CCR, CAR, and friends

Peter Otte

September 10, 2007

Contents

1 Introduction **introduction** **3**

2 Preliminaries **preliminaries** **3**

3 CCR **ccr** **8**

4 CAR **car** **10**

5 CCR – Fock case **ccr_fock** **11**

6 CAR – Fock case **car_fock** **16**

7 Vertex operators **17**

8 Free second quantization – bosons **second_quantization** **18**

8.1 The number operator second_quantization_number 19

8.2 General quadratic operators second_quantization_general 23

8.3 Quadratic operators second_quantization_quadratic 26

8.4 Exponentials of operators second_quantization_exponentials 33

9 Second quantization – Fermions **40**

9.1 The fermionic number operator 41

9.2 General fernionic operators 46

9.3 Quadratic fermionic operators 49

9.4 Exponentials of fermionic operators 56

10 Appendix Second quantization – Fermions **63**

11 Bogoliubov transforms bogoliubov **63**

11.1 Bosons bogoliubov_b 63

11.2 Reduction to normal forms – bosons 71

11.2.1 An example bogoliubov_ex_b 71

11.3 Fermions bogoliubov_f 73

11.4 Appendix: The kernel of a Bogoliubov operator bogoliubov_f_appendix 77

11.5 Inhomogeneous fermionic Bogoliubov transforms bogoliubov_inh_f 79

11.6 Inhomogeneous bosonic Bogoliubov transforms bogoliubov_inh_b 79

11.7 Reduction to normal forms - Fermions 79

12 Quasi-free representation of CCR quasi_free_b **79**

13 Quasi-free representation of CAR quasi_free_f **81**

14 Equivalence of quasi-free representations quasi_free_equivalence **82**

15 Hard core bosons hard_core_bosons **82**

16 Multi boson operators multi_boson **83**