

Anderson's Orthogonality Catastrophe (AOC)

Peter Otte

Ruhr-Universität Bochum

joint work with
H. Küttler (Munich) and W. Spitzer (Hagen)
supported by
SFB-TR 12/P. Müller (Munich)

ICMP12
Aalborg, August 2012

The Problem

Physics

- ▶ N non-interacting **fermions** in $[-L, L]^d$, here $d = 1$
- ▶ Sudden perturbation (greatly simplified, no time evolution)
- ▶ Overlap between old and new ground state?

Mathematics

- ▶ One particle operators

$H_0 = -\Delta$, potential V , $H := H_0 + V$, Hilbert space $\mathcal{L}^2[-L, L]$

- ▶ φ_j^L, ψ_j^L normalized Dirichlet eigenvectors of H_0, H , respectively
- ▶ No interaction: overlap is given by a **Slater determinant**

$$\mathcal{D}_N^L := \det((\varphi_j^L, \psi_k^L))_{j,k=1,\dots,N}$$

AOC

Anderson inequality - Hadamard, Bessel, etc.

$$|\det((\varphi_j^L, \psi_k^L))|^2 \leq \exp \left[- \sum_{j=1}^N \sum_{k=N+1}^{\infty} |(\varphi_j^L, \psi_k^L)|^2 \right] =: e^{-\mathcal{I}_N^L}$$

Asymptotics in the thermodynamic limit

Anderson (1967) derived informally

$$\mathcal{I}_N^L \approx \gamma \ln N, \quad \gamma > 0$$

where $N, L \rightarrow \infty$ with constant particle density N/L

Orthogonality catastrophe

$$|\mathcal{D}_N^L|^2 \approx N^{-\gamma} \rightarrow 0$$

Mathematical Tools

Integral formula - Riesz, Krein, etc.

$R_0^L(z)$ free resolvent, Γ_N appropriate contour of integration, P_N^L projection onto $\text{span}\{\varphi_1^L, \dots, \varphi_N^L\}$

$$\mathcal{I}_N^L = \frac{1}{2\pi i} \int_{\Gamma_N} \text{tr}[(\mathbb{1} - VR_0^L(z))^{-1} V P_N^L R_0^L(z) (\mathbb{1} - VR_0^L(z))^{-1} V (R_0^L(z))^2] dz$$

Asymptotics

$P_N^L R_0^L(z)$ will produce the **logarithm**

Delta sequence

$(R_0^L(z))^2$ produces a **delta function** in the thermodynamic limit

$$(R_0^L(z; x, y))^2 \rightsquigarrow D_N^L(z) = \frac{L}{\sin^2(L\sqrt{z})}, \quad z \in \Gamma_N$$

Asymptotic Formula $N, L \rightarrow \infty$

Delta sequence argument

\mathcal{I}_N^L can be evaluated at a point ν_N^L . With some nice operator F_N^L

$$\mathcal{I}_N^L \sim \text{tr}(V P_N^L R_0^L(\nu_N^L) F_N^L), \quad \nu_N^L := \left(\frac{\pi}{2L} \left(N + \frac{1}{2}\right)\right)^2$$

Logarithm

With the free eigenvalues $\lambda_j^L = (\pi j / (2L))^2$

$$P_N^L R_0^L(\nu_N^L) = \sum_{j=1}^N \frac{1}{\nu_N^L - \lambda_j^L} (\varphi_j^L, \cdot) \varphi_j^L \rightsquigarrow \sum_{j=1}^N \frac{1}{N + \frac{1}{2} - j} = \ln N + O(1)$$

Conditions on the potential V

Compact support (essential) and some technical properties