How to express in a formally precise way modifiers?

We have a proposition like “She bakes a cake” and put it within the scope of a modifier, e.g.:

- Possibly/Necessarily, she bakes a cake.
- It should be that she bakes a cake.
- I know/believe that she bakes a cake.

idea: accessible worlds

accessibility relates to the modifier in question

- accessible worlds are possible worlds
- accessible worlds are ethically/legally/etc. ideal worlds
- accessible worlds are states/worlds compatible with my doxastic/epistemic state

How to define an entailment relation? The modal logic $\mathbf{K}$

- Answer: as usual!
- only: where $M = \langle W, R, v \rangle$: $M \models A$ iff for all $w \in W$, $M, w \models A$.
- We say $M$ is a model of $\Gamma$ iff $M \models A$ for all $A \in \Gamma$
- $\Gamma \models A$ iff all models of $\Gamma$ are models of $A$
- often the semantics is phrased with a so-called actual/designated world
  - a model is a quadruple $M = \langle W, R, v, a \rangle$
  - difference to before: $M \models A$ iff $M, a \models A$.
  - rest as before: $\Gamma \models A$ iff all models of $\Gamma$ are models of $A$
- these two representational formats are semi-expressive/equivalent: try to show this!

What do you think: $\Box A \vdash^K \Diamond A$?

Show: $\Box (A \lor B) \vdash^K \Box (A \land B)$

Axiomatizing $\mathbf{K}$

$\mathbf{N}$ $\vdash A$ implies $\vdash \Box A$

$\mathbf{K}$ $\vdash (\Box (A \lor B) \lor (\Box A \land \Box B)$

- Show: $\Box (A \lor B) \vdash^K \Box A \land \Box B$
- Show: $\Box A, \Box B \vdash^K \Box (A \land B)$
- Show: $\Box A \lor \Box B, \Box \neg B \vdash^K \Box A$
Strengthening Kripkean Semantics: Frame Conditions

- requiring $R$ to be reflexive (i.e., $(w, w) \in R$ for all $w$): $\text{T} \models K + \Box A \supset A'$
- requiring $R$ to be serial (i.e., for all $w$ there is a $w'$ such that $(w, w') \in R$): $\text{KD} \models K + \Box A \supset \Box A'$
- requiring that the accessibility relation $\leq$ is a partial order (reflexivity, transitivity, antisymmetry): intuitionistic logic
- additional requirement: $w \leq w'$ implies $v(w) \leq v(w')$
- negation is a modal operator: $M, w \models \neg A$ iff there is no $w'$ such that $(w, w') \in R$ and $w' \models A$
- implication: $M, w \models A \rightarrow B$ iff for all $w'$ for which $(w, w') \in R$, $M, w' \models A$ implies $M, w \models B$.

**Standard Deontic Logic is KD.**

Deontic Logic Paradox

- Natural language representation: NR
- formal representation: FR
- Paradox 1:
  - NR implies $A$
  - FR does not imply $A$
- Paradox 2:
  - FR implies $A$.
  - NR does not imply $A$.

Deontic Dilemmas

Example: The dilemma of Sartre’s pupil

- Obligation 1: stay with the ill mother
- Obligation 2: join the forces to fight the Nazis

Formal definition

- Two obligations: $OA$, $OB$
  - both are possible: $\Diamond A$, $\Diamond B$
  - they cannot jointly be realized: $\neg \Diamond (A \land B)$

They are often characterized by

- obligations with equal force
- incommensurable obligations

Deontic Explosions

**Conflict (A, B) ⊢ ⊥**

Example 1: Principle D

| D | OA → ¬O¬A |
| ECQ | $A \land \neg A \rightarrow \bot$ |

| 1 | OA |
| 2 | $O\neg A$ |
| 3 | $OA \rightarrow \neg O\neg A$ | D |
| 4 | $\neg O\neg A$ | 1, 3; MP |
| 5 | $O\neg A \land \neg O\neg A$ | 2, 4; $\land$ intro |
| 6 | ⊥ | 5; ECQ |

Example 2: Aggregation and Kant’s “ought implies can”

| AND | OA ∧ OB → O(A ∧ B) |
| ECQ | $A \land \neg A \rightarrow \bot$ |

| 1 | OA |
| 2 | $O\neg A$ |
| 3 | $OA \rightarrow \neg O\neg A$ | D |
| 4 | $\neg O\neg A$ | 1, 3; MP |
| 5 | $O\neg A \land \neg O\neg A$ | 2, 4; $\land$ intro |
| 6 | ⊥ | 5, 6; ECQ |

Deontic Explosions
Deontic Explosions

1 OA
Starting point: 2 OB
3 ¬♦(A ∧ B)

Example 3: Distribution

\[
\text{RM} \quad □(A → B) → (OA → OB)
\]
\[
\text{D} \quad OA → ¬O¬A
\]
\[
\text{ECQ} \quad A ∧ ¬A → ⊥
\]

4 □(A → ¬B) 3
5 O¬B 1, 4; RM
6 ¬O¬B 2; D
7 ⊥ 5, 6; ECQ

Many “Bad” Combinations

\[
\text{AND} \quad OA ∧ OB → O(A ∧ B)
\]
\[
\text{NM} \quad □(A → B) → (OA → OB)
\]

\[
\text{KP} \quad OA → O\text{A}
\]
\[
\text{ECQ} \quad A ∧ ¬A → ⊥
\]
\[
\text{D} \quad OA → ¬O¬A
\]

Approaches for logics dealing with deontic explosions:

- Restricting/Rejecting ECQ — going paraconsistent
  - Da Costa&Carnielli (1986) [4], Beirlaen et al [3, 2, 1]
  - Restricting AND: Goble’s logic P
  - Restricting RM: Goble’s logics DPM

Restricting Aggregation [7, 8]

- preferential semantics:
  - \( M = (W, (\leq_s)_{s \in W}, v) \) where \( \leq_s \) are preorder on their fields (reflexive and transitive)
  - the field of a relation: \( F \leq_s = \{ b \in W \mid \text{there is a } c \in W \text{ such that } b \leq_s c \text{ or } c \leq_s b \} \)
  - define: \( M, w \models OA \text{ iff there is a } w' \in F \leq_w \text{ such that for all } w'' \text{ for which } w' \leq_{w''} w'' \), \( M, w'' \models A \)
  - where \( \leq_s \) are also connected (for all \( w \neq w' \) in \( F \leq_s, w \leq w' \) or \( w' \leq w \)): this semantics characterizes SDL (and as we will see below also a dyadic version of SDL)
  - multi-relational semantics (generalization of Kripkean semantics)
    - \( M = (W, R, v) \) where \( R \) is a non-empty family of serial accessibility relations \( R \)
    - each \( R \) represents a normative standard/value system/etc.: \( M, w \models OA \text{ iff there is a } R \in \mathcal{R} \text{ such that } M, w \models A \text{ for all } w' \in W \text{ for which } (w, w') \in R \)
    - both systems characterize the same consequence relation

Some other approaches

  - argumentation theory: Oren et al. (2008) [14], Gabbay (2012) [6], Straßer&Arieli (2014) [18]

System P

\[
\text{PC} \quad \text{If } A \text{ is a classical tautology then } A \text{ is an axiom of } P
\]
\[
\text{RM} \quad \text{If } \models A ⊃ B \text{ then } \models OA ⊃ OB
\]
\[
\text{N} \quad \models O\top
\]
\[
\text{P} \quad \models ¬O¬\bot
\]
Restricting Inheritance

Replace the inheritance principle
\[ \text{RM: } \text{if } \vdash A \rightarrow B \text{ then } \vdash O A \rightarrow O B \]
by a restricted version:
\[ \text{RPM: } \text{if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB) \]

Goble’s Logic DPM.1

DPM.1 Axioms
all axioms of classical propositional calculus and
\[ \text{RPM: } \text{if } \vdash A \rightarrow B \text{ then } \vdash PA \rightarrow (OA \rightarrow OB) \]
\[ \text{RE: } \text{if } \vdash A \leftrightarrow B \text{ then } \vdash OA \leftrightarrow OB \]
\[ \text{N: } \vdash O \top \]
\[ \text{AND: } \vdash (OA \land OB) \rightarrow O(A \land B) \]

Semantics

Neighborhood Frame
A neighborhood frame \( F \) is a pair \( \langle W, O \rangle \) in which \( W \) is a non-empty set of points, e.g., possible worlds, and \( O \) is a function assigning every \( a \in W \) a set, \( O_a \), of subsets of \( W \); i.e., \( O_a \subseteq \wp W \).

Models
A model, \( M \), is a pair \( \langle F, v \rangle \) where \( F \) is a neighborhood frame \( \langle W, O \rangle \), and \( v \) is a function assigning every atomic formula \( p \) of \( L \) a subset of \( W \); i.e., \( v(p) \subseteq W \). A satisfaction relation \( \models \) is defined as follows:
\[
\begin{align*}
\text{Tp: } & M, a \models p \text{ iff } a \in v(p) \\
\text{T¬: } & M, a \models \neg A \text{ iff } M, a \not\models A \\
\text{T∧: } & M, a \models A \land B \text{ iff } M, a \models A \text{ and } M, a \models B \\
\text{T∨: } & M, a \models A \lor B \text{ iff } M, a \models A \text{ or } M, a \models B \\
\text{TO: } & M, a \models OA \text{ iff } |A|_M \in O_a
\end{align*}
\]

Problem with the Weakenings of SDL

- they are weak!
- solution: adaptive strengthening
- e.g., in the context of \( P \): apply aggregation conditionally as much as possible
- e.g., in the context of \( \text{DPM} \): apply inheritance conditionally and as much as possible

Asparagus – Specificity

- Being served a meal, you ought not to eat with fingers.
- Being served asparagus, you ought to eat with fingers.
- You’re being served asparagus.

What if we model this via classical implication?

- \( m \supset O \neg f \)
- \( (m \land a) \supset Of \)
- \( m \land a \)

Problem: This is classically inconsistent (if SDL models \( O \)).
Problems with Conditional Obligations 2

Chisholm’s Paradox: Contrary to Duty Obligations

- John ought not to impregnate Suzy Mae.
- If John impregnates Suzy Mae, he ought to marry her.
- If John doesn’t impregnate Suzy Mae, he ought not to marry her.
- John impregnates Suzy Mae.

Desiderata of a Formal Modeling

- logical independence
- non-triviality
- symmetry/non-ad-hoc modeling
- detachment possible

Chisholm continued

Option 1

- \( \top \supset \neg i \)
- \( i \supset m \)
- \( \neg i \supset \neg m \)
- \( i \)

What’s the problem?

Option 2

- \( \top \supset \neg i \)
- \( i \supset m \)
- \( \neg i \supset \neg m \)
- \( i \)

What’s the problem?

Some Approaches to Conditional Obligations

- use of binary modal operators
- default logic approach
- Input/Output logic

Chisholm continued

Option 3

- \( \top \supset \neg i \)
- \( i \supset m \)
- \( \neg i \supset \neg m \)
- \( i \)

What’s the problem?

Option 4

- \( \top \supset \neg i \)
- \( i \supset m \)
- \( \neg i \supset \neg m \)
- \( i \)

What’s the problem?

Using preferential semantics for a dyadic version of SDL

- \( M = \langle W, \leq, v \rangle \) where \( \leq \) is a connected preorder (reflexive and transitive)
- \( M, w \models O(B/A) \) iff there is a \( w' \in F \leq w \) such that \( M, w' \models A \land B \) and for any \( w'' \) for which \( w' \leq w'' \): \( M, w'' \models A \) implies \( M, w'' \models B \).
- axiomatized by:
  - RCE If \( \vdash A \equiv A' \) then \( \vdash O(B/A) \equiv O(B/A') \)
  - RCM If \( \vdash B \equiv C \) then \( \vdash O(B/A) \equiv O(C/A) \)
  - CK \( O(B/C/A) \equiv O(B/A) \land O(C/A) \)
  - CD \( O(B/A) \equiv \neg O(\neg B/A) \)
  - CN \( O(\top/\top) = O(B/A) \equiv O(A \land B/A) \)
  - trans \( \langle (A \geq B) \land (B \geq C) \rangle \equiv (A \geq C) \) where

\[ A \geq B = df \neg O(\neg A/A \lor B) \] (read: “A is at least as good as B”)

SDDL and a Problem with Specificity

Problem: we get Rational Monotonicity:

\[ \text{RM} \ (O(A/B) \land P(C/B)) \supset O(A/B \land C) \]

\[ \text{e.g., } (O(\neg f/m) \land P(a/m)) \supset O(\neg f/m \land a) \]

- Various solutions have been proposed to weaken the monotonicity principle further and using different semantics (such as neighborhood semantics).
- All that I know generate other problematic examples (see [17, Part IV])
- One option: instead of “hard-coding” a weakened monotonicity principle, “go adaptive”. I.e., apply monotonicity \( O(A/B) \equiv O(A/B \land C) \) defeasibly “as much as possible”. 
The Detachment Problem

- Van Eck: “How can we take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional obligation?” [21]
- SDDL has not means for detachment
- general problem: the possibility of specificity cases and CTD-cases means that we cannot apply MP naively to conditional obligations
- how to deal with this problem then? E.g.,
  - Input/Output logic (Makinson / Van Der Torre)
  - Adaptive logics (the Gent crew)
  - Default Logic (Horty)

Handwritten notes:

- Christian Straßer
  - A system of temporally relative modal and deontic predicate logic and its philosophical applications.
    Published online first.
- John F. Horty
  - Reasoning with moral conflicts.
- Lou Goble
  - Bipolar argumentation frames and contrary to duty obligations, preliminary report.
- Lou Goble
  - Multiplex semantics for deontic logic.
- Lou Goble
- Lou Goble
  - Normative conflicts and the logic of ‘ought’.
- Lou Goble
  - Deontic logic (adapted) for normative conflicts.

Handwritten notes:

- Dov M. Gabbay
  - Bipolar argumentation frames and contrary to duty obligations.
- Dov M. Gabbay
  - Bipolar argumentation frames and contrary to duty obligations, preliminary report.
- Lou Goble
- Lou Goble
  - Normative conflicts and the logic of ‘ought’.
- Lou Goble
  - Deontic logic (adapted) for normative conflicts.

Handwritten notes:

- Mathieu Beirlaen and Christian Straßer
  - Nonmonotonic reasoning with normative conflicts in multi-agent deontic logic.
- Mathieu Beirlaen and Christian Straßer
  - Two adaptive logics of norm-propositions.
- Mathieu Beirlaen, Christian Straßer, and Joke Meheus
  - An inconsistency-adaptive deontic logic for normative conflicts.
- Newton C.A. da Costa and Walter Carnielli
  - On paraconsistent deontic logic.
    - Philosophy, 16:293–305, 1986.
- Bas C. Van Fraassen
  - Values and the heart’s command.