Normative Reasoning and Deontic Logics
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How to express in a formally precise way modifiers?

We have a proposition like “She bakes a cake” and put it within the scope of a modifier, e.g.:

- Possibly/Necessarily, she bakes a cake.
- It should be that she bakes a cake.
- I know/believe that she bakes a cake.

idea: accessible worlds

accessibility relates to the modifier in question

- accessible worlds are possible worlds
- accessible worlds are ethically/legally/etc. ideal worlds
- accessible worlds are states/worlds compatible with my doxastic/epistemic state
Modal Logics – Kripkean Frames, More Formally

- a model is a tuple $M = \langle W, R, v \rangle$ where
- $W$ is a set of points often called “worlds”
- $R \subseteq W \times W$ is a relation often called “accessibility relation”
- $v : W \times \mathcal{A} \to \{0, 1\}$ is an assignment function
  - often also: $v : \mathcal{A} \to \mathcal{P}(W)$ or $v : W \to \mathcal{P}(\mathcal{A})$
- atoms get their truth value at each world $w \in W$ in a model $M$ via the assignment just as expected:
  $M, w \models A$ (where $A \in \mathcal{A}$) iff $v(w, A) = 1$ (resp. iff $w \in v(A)$, resp. iff $A \in v(w)$)
- the classical connectives are interpreted at each world in a model just as expected:
  $M, w \models \neg A$ iff $M, w \not\models A$
  $M, w \models A \land B$ iff $M, w \models A$ and $M, w \models B$
  $M, w \models A \lor B$ iff $M, w \models A$ or $M, w \models B$
  etc.
we now also have a unary modal operator □: 
\[ M, w \models □A \text{ iff for all } w' \text{ for which } (w, w') \in R \text{ (read: “for all worlds } w' \text{ accessible from } w): } M, w' \models A \]
e.g., \( M, w \models □A \) where □ represents necessity means that \( A \) holds in all possible worlds
e.g., \( M, w \models □A \) where □ represents normativity means that \( A \) holds in all ideal worlds

etc.

duality principle: ◊ = ¬□¬

◊ represents possibility

◊ represents permission

etc.

truth for ◊:
\[ M, w \models ◊A \text{ iff there is a } w' \text{ such that } (w, w') \in R \text{ and } M, w' \models A \]

What do you think: \( M, w \models □A \) implies \( M, w \models ◊A \)?

Show: \( M, w \models □(A \land B) \) implies \( M, w \models □A \land □B \)

Show: \( M, w \models □A, □B \) implies \( M, w \models □(A \land B) \)
How to define an entailment relation? The modal logic $\textbf{K}$

- Answer: as usual!
- only: where $M = \langle W, R, v \rangle$: $M \models A$ iff for all $w \in W$, $M, w \models A$.
- We say $M$ is a model of $\Gamma$ iff $M \models A$ for all $A \in \Gamma$
- $\Gamma \vdash A$ iff all models of $\Gamma$ are models of $A$
- often the semantics is phrased with a so-called actual/designated world
  - a model is a quadruple $M = \langle W, R, v, a \rangle$
  - difference to before: $M \models A$ iff $M, a \models A$.
  - rest as before: $\Gamma \vdash A$ iff all models of $\Gamma$ are models of $A$
- these two representational formats are semi-expressive/equivalent: try to show this!

- What do you think: $\Box A \vdash_{\textbf{K}} \Diamond A$?
- Show: $\Box (A \land B) \vdash_{\textbf{K}} \Box A \land \Box B$
- Show: $\Box A, \Box B \vdash_{\textbf{K}} \Box (A \land B)$
Axiomatizing $\mathbf{K}$

$\mathbf{N} \vdash A$ implies $\vdash \Box A$

$\mathbf{K} \vdash \Box (A \supset B) \supset (\Box A \supset \Box B)$

- Show: $\Box (A \land B) \vdash_{\mathbf{K}} \Box A \land \Box B$
- Show: $\Box A, \Box B \vdash_{\mathbf{K}} \Box (A \land B)$
- Show: $\Box A, \Diamond B \vdash_{\mathbf{K}} \Diamond (A \land B)$
- Show: $\Box A \lor \Box B, \Diamond \neg B \vdash_{\mathbf{K}} \Box A$
Strengthening Kripkean Semantics: Frame Conditions

- requiring $R$ to be reflexive (i.e., $(w, w) \in R$ for all $w$): $\text{T} \ "= K + \vdash \Box A \supset A$"
- requiring $R$ to be serial (i.e., for all $w$ there is a $w'$ such that $(w, w') \in R$): $\text{KD} \ "= K + \vdash \Box A \supset \Diamond A$"
- requiring that the accessibility relation $\leq$ is a partial order (reflexivity, transitivity, antisymmetry): intuitionistic logic
  - additional requirement: $w \leq w'$ implies $v(w) \subseteq v(w')$
  - negation is a modal operator: $M, w \models \neg A$ iff there is no $w'$ such that $(w, w') \in R$ and $w' \models A$
  - implication: $M, w \models A \rightarrow B$ iff for all $w'$ for which $(w, w') \in R$, $M, w' \models A$ implies $M, w \models B$.

**Standard Deontic Logic** is $\text{KD}$. 
Deontic Logic Paradox

- Natural language representation: NR
- Formal representation: FR
- Paradox 1:
  - NR implies A
  - FR does not imply A
- Paradox 2:
  - FR implies A.
  - NR does not imply A.
Some Problems

- \( \vdash A \supset B \) implies \( \vdash OA \supset OB \)
  - Ross' Paradox: \( O\ell \) (you're supposed to post the letter) implies \( O(\ell \lor b) \) (you're supposed to post the letter or burn the city)
  - The birthday cake: \( O(i_1 \land \ldots \land i_n) \) implies \( Oi_j \) (1 ≤ j ≤ n)?

- problems with conditional obligations
  - specificity: \( m \supset O\neg f \) but \( (m \land a) \supset Of \). Suppose \( m \). Then we get triviality in SDL
  - similar: contrary-to-duty (Forrester paradox, Chisholm paradox, see later)

- deontic conflicts: e.g., \( OA \land O\neg A \)
Deontic Dilemmas

Example: The dilemma of Sartre’s pupil

- Obligation 1: stay with the ill mother
- Obligation 2: join the forces to fight the Nazis

Formal definition

- Two obligations: $OA$, $OB$
- both are possible: $\Diamond A$, $\Diamond B$
- they cannot jointly be realized: $\neg \Diamond (A \land B)$

They are often characterized by

- obligations with equal force
- incommensurable obligations
Deontic Explosions

## Conflict \((A, B) \vdash \bot\)

### Example 1: Principle \(D\)

\[
\begin{align*}
D & \quad OA \rightarrow \neg O \neg A \\
ECQ & \quad A \land \neg A \rightarrow \bot
\end{align*}
\]

<table>
<thead>
<tr>
<th>Step</th>
<th>Formula</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>OA</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>O\neg A</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>OA \rightarrow \neg O \neg A</td>
<td>D</td>
</tr>
<tr>
<td>4</td>
<td>\neg O \neg A</td>
<td>1, 3; MP</td>
</tr>
<tr>
<td>5</td>
<td>O\neg A \land \neg O \neg A</td>
<td>2, 4; \land \neg intro</td>
</tr>
<tr>
<td>6</td>
<td>\bot</td>
<td>5; ECQ</td>
</tr>
</tbody>
</table>
Deontic Explosions

1  $OA$

Starting point: 2  $OB$

3  $\neg \lozenge (A \land B)$

Example 2: Aggregation and Kant’s “ought implies can”

\[
\begin{array}{|c|c|}
\hline
\text{AND} & OA \land OB \rightarrow O(A \land B) \\
\text{Kant’s principle} & OA \rightarrow \lozenge A \\
\text{ECQ} & A \land \neg A \rightarrow \bot \\
\hline
\end{array}
\]

4  $O(A \land B)$  1, 2; AND

5  $\lozenge (A \land B)$  3, 4; Kant’s principle

6  $\bot$  5, 6; ECQ
Deontic Explosions

1  $OA$

Starting point: 2  $OB$

3  $\neg \Diamond (A \land B)$

Example 3: Distribution

<table>
<thead>
<tr>
<th>Rule</th>
<th>Premise</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM</td>
<td>$\Box (A \rightarrow B) \rightarrow (OA \rightarrow OB)$</td>
</tr>
<tr>
<td>D</td>
<td>$OA \rightarrow \neg O \neg A$</td>
</tr>
<tr>
<td>ECQ</td>
<td>$A \land \neg A \rightarrow \bot$</td>
</tr>
</tbody>
</table>

4  $\Box (A \rightarrow \neg B)$  3

5  $O \neg B$  1, 4; RM

6  $\neg O \neg B$  2; D

7  $\bot$  5, 6; ECQ
DEX0  Conflict implies triviality (anything follows)
DEX1  Conflict implies that anything is obligatory

- Show that AND and $\vdash \neg O \bot$ implies DEX0
- Show that AND and RM' (see below) implies DEX1
  RM'  If $\vdash A \supset B$ then $\vdash OA \supset OB$. 
Many “Bad” Combinations

\[\textbf{AND}\]
\[OA \land OB \rightarrow O(A \land B)\]

\[\textbf{NM}\]
\[\square(A \rightarrow B) \rightarrow (OA \rightarrow OB)\]

\[\textbf{KP}\]
\[OA \rightarrow \diamond A\]

\[\textbf{ECQ}\]
\[A \land \neg A \rightarrow \bot\]

\[\textbf{D}\]
\[OA \rightarrow \neg O \neg A\]

Approaches for logics dealing with deontic explosions:

- Restricting/Rejecting \textbf{ECQ} – going paraconsistent
  - Da Costa&Carnielli (1986) [4], Beirlaen et al [3, 2, 1]
- Restricting \textbf{AND}: Goble’s logic \(\mathcal{P}\)
- Restricting \textbf{RM}: Goble’s logics \(\text{DPM}\)
Some other approaches

- argumentation theory: Oren et al. (2008) [14], Gabbay (2012) [6], Straßer & Arieli (2014) [18]
Restricting Aggregation [7, 8]

- preferential semantics:
  - $M = \langle W, \langle \leq_a \rangle_{a \in W}, v \rangle$ where $\leq_a$ are preorders on their fields (reflexive and transitive)
  - the field of a relation: $\mathcal{F} \leq_a = \{ b \in W \mid$ there is a $c \in W$ such that either $b \leq_a c$ or $c \leq_a b \}$
  - define: $M, w \models OA$ iff there is a $w' \in \mathcal{F} \leq_w$ such that for all $w''$ for which $(w', w'') \in R$, $M, w'' \models A$
  - where $\leq_a$ are also connected (for all $w \neq w'$ in $\mathcal{F} \leq_a$, $w \leq w'$ or $w' \leq w$): this semantics characterizes SDL (and as we will see below also a dyadic version of SDL)

- multi-relational semantics (generalization of Kripkean semantics)
  - $M = \langle W, R, v \rangle$ where $R$ is a non-empty family of serial accessibility relations $R$
  - each $R$ represents a normative standard/value system/etc.:
  - $M, w \models OA$ iff there is a $R \in \mathcal{R}$ such that $M, w \models A$ for all $w' \in W$ for which $(w, w') \in R$

- both systems characterize the same consequence relation
System $\mathbf{P}$

**PC** If $A$ is a classical tautology then $A$ is an axiom of $\mathbf{P}$

**RM** If $\vdash A \supset B$ then $\vdash OA \supset OB$

**N** $\vdash OT$

**P** $\vdash \neg O \neg T$
Restricting Inheritance

Replace the inheritance principle

\[ \text{RM} \quad \text{if} \; \vdash A \to B \quad \text{then} \quad \vdash OA \to OB \]

by a restricted version:

\[ \text{RPM} \quad \text{if} \; \vdash A \to B \quad \text{then} \quad \vdash PA \to (OA \to OB) \]
DPM.1 Axioms

all axioms of classical propositional calculus and

RPM if $\vdash A \rightarrow B$ then $\vdash PA \rightarrow (OA \rightarrow OB)$

RE if $\vdash A \leftrightarrow B$ then $\vdash OA \leftrightarrow OB$

N $\vdash O\top$

AND $\vdash (OA \land OB) \rightarrow O(A \land B)$
Semantics

Neighborhood Frame

A *neighbourhood frame* $F$ is a pair $\langle W, O \rangle$ in which $W$ is a non-empty set of points, e.g., possible worlds, and $O$ is a function assigning every $a \in W$ a set, $O_a$, of subsets of $W$; i.e., $O_a \subseteq \wp W$.

Models

A *model*, $M$, is a pair $\langle F, v \rangle$ where $F$ is a neighborhood frame $\langle W, O \rangle$, and $v$ is a function assigning every atomic formula $p$ of $\mathcal{L}$ a subset of $W$, i.e., $v(p) \subseteq W$. A *satisfaction relation* $\models$ is defined as follows.

- $T_p)$ $M, a \models p$ iff $a \in v(p)$
- $T\neg)$ $M, a \models \neg A$ iff $M, a \not\models A$
- $T\land)$ $M, a \models A \land B$ iff $M, a \models A$ and $M, a \models B$
- $T\lor)$ $M, a \models A \lor B$ iff $M, a \models A$ or $M, a \models B$
- $T_O)$ $M, a \models O_A$ iff $A|_M \in O_a$
Semantics

For DPM.1: For all $X, Y \subseteq W$ and all $a \in W$:

a) $W \in O_a$

b) If $X \in O_a$ and $Y \in O_a$ then $X \cap Y \in O_a$

c) If $X \subseteq Y$ and $X \in O_a$ and $-X \notin O_a$ then $Y \in O_a$

Condition a) validates N, condition b) validates AND and condition c) validates RPM.
Problem with the Weakenings of **SDL**

- they are weak!
- solution: adaptive strengthening
- e.g., in the context of **P**: apply aggregation conditionally as much as possible
- e.g., in the context of **DPM**: apply inheritance conditionally and as much as possible
Problems with Conditional Obligations

Asparagus – Specificity

- Being served a meal, you ought not to eat with fingers.
- Being served asparagus, you ought to eat with fingers.
- You’re being served asparagus.

What if we model this via classical implication?

- \( m \supset O \neg f \)
- \( (m \land a) \supset Of \)
- \( m \land a \)

**Problem:** This is classically inconsistent (if SDL models \( O \)).
Chisholm’s Paradox: Contrary to Duty Obligations

- John ought not to impregnate Suzy Mae.
- If John impregnates Suzy Mae, he ought to marry her.
- If John doesn’t impregnate Suzy Mae, he ought not to marry her.
- John impregnates Suzy Mae.

Desiderata of a Formal Modeling

- logical independence
- non-triviality
- symmetry/non-ad-hoc modeling
- detachment possible
Chisholm continued

Option 1

- $\top \supset O\neg i$
- $i \supset Om$
- $\neg i \supset O\neg m$
- $i$

What’s the problem?

Option 2

- $O(\top \supset \neg i)$
- $O(i \supset m)$
- $O(\neg i \supset \neg m)$
- $i$

What’s the problem?
Chisholm continued

Option 3

- $O(\top \supset \neg i)$
- $O(i \supset m)$
- $\neg i \supset O\neg m$
- $i$

What’s the problem?

Option 4

- $O(\top \supset \neg i)$
- $i \supset Om$
- $O(\neg i \supset \neg m)$
- $i$

What’s the problem?
Some Approaches to Conditional Obligations

- use of binary modal operators
- default logic approach
- Input/Output logic
Using preferential semantics for a dyadic version of **SDL**

- \( M = \langle W, \leq, v \rangle \) where \( \leq \) is a connected preorder (reflexive and transitive)

- \( M, w \models O(B/A) \) iff there is a \( w' \in F \leq_w \) such that \( M, w' \models A \land B \) and for any \( w'' \) for which \( w' \leq_w w'' \):
  \( M, w'' \models A \) implies \( M, w'' \models B \).

- axiomatized by:
  - **RCE** If \( \vdash A \equiv A' \) then \( \vdash O(B/A) \equiv O(B/A') \)
  - **RCM** If \( \vdash B \supset C \) then \( \vdash O(B/A) \supset O(C/A) \)
  - **CK** \( O(B \supset C/A) \supset (O(B/A) \supset O(C/A)) \)
  - **CD** \( O(B/A) \supset \neg O(\neg B/A) \)
  - **CN** \( O(\top/\top) \)
  - **CO\land** \( O(B/A) \supset O(A \land B/A) \)
  - **trans** \( (A \geq B) \land (B \geq C) \supset (A \geq C) \) where

\[
A \geq B =_{\text{df}} \neg O(\neg A/A \lor B) \quad \text{(read: "A is at least as good as B")}
\]
SDDL and a Problem with Specificity

Problem: we get Rational Monotonicity:

\[ \text{RM} \quad (O(A/B) \land P(C/B)) \supset O(A/B \land C) \]

\[ \text{e.g., } (O(\neg f/m) \land P(a/m)) \supset O(\neg f/m \land a) \]

- Various solutions have been proposed to weaken the monotonicity principle further and using different semantics (such as neighborhood semantics).
- All that I know generate other problematic examples (see [17, Part IV])
- One option: instead of “hard-coding” a weakened monotonicity principle, “go adaptive”. I.e., apply monotonicity \((O(A/B) \supset O(A/B \land C))\) defeasibly “as much as possible”.
The Detachment Problem

- Van Eck: “How can we take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional obligation?” [21]
- SDDL has not means for detachment
- general problem: the possibility of specificity cases and CTD-cases means that we cannot apply MP naively to conditional obligations
- how to deal with this problem then? E.g.,
  - Input/Output logic (Makinson / Van Der Torre)
  - Adaptive logics (the Gent crew)
  - Default Logic (Horty)


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Dov M. Gabbay.
Bipolar argumentation frames and contrary to duty obligations, preliminary report.

Lou Goble.
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Lou Goble.

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John F. Hory.
Reasoning with moral conflicts.

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Constraints for Input/Output logics.

Joke Meheus, Mathieu Beirlaen, and Frederik Van De Putte.
Avoiding deontic explosion by contextually restricting aggregation.

An argumentation inspired heuristic for resolving normative conflict.
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An adaptive logic framework for conditional obligations and deontic dilemmas.

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