Reasoning by Cases in the Nonmonotonic Wilderness

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Outline

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Reasoning by Cases in Structured Argumentation

Complications

Weak Contraposition

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Introduction
Reasoning by Cases in Structured Argumentation.

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What we expected ...
... and how it turned out ...
What is Reasoning by Cases?
Reasoning by Cases

• in natural deduction

\[
\frac{\begin{array}{c}
\Gamma \\
\Gamma, [A]^1 \\
\Gamma, [B]^1 \\
A \lor B \\
C \\
C
\end{array}}{C \lor E}\]
Reasoning by Cases

• in natural deduction

\[ \begin{align*}
\Gamma & \quad \Gamma, [A]^1 & \quad \Gamma, [B]^1 \\
\vdots & \quad \vdots & \quad \vdots \\
A \lor B & \quad C & \quad C \\
\hline \\
C & \quad & \lor E_1
\end{align*} \]

• expressed with \( \sqcup \):

\[ A \lor B \quad A \sqcup C \quad B \sqcup C \\
\hline \\
C \]
Reasoning by Cases, Defeasibly

- strict rules ("\(\rightarrow\)\)) vs. defeasible rules ("\(\Rightarrow\)\))
Reasoning by Cases, Defeasibly

• strict rules (“→”) vs. defeasible rules (“⇒”)
• schematically:

\[
\frac{A \lor B}{A \Rightarrow C \quad B \Rightarrow C} \quad \therefore C
\]
Reasoning by Cases, Defeasibly

• strict rules (“→”) vs. defeasible rules (“⇒”)
• schematically:

\[
\frac{A \lor B \quad A \Rightarrow C \quad B \Rightarrow C}{C}
\]

• or, more generally:

\[
\frac{A \lor B \quad A \Rightarrow \cdots \Rightarrow C \quad B \Rightarrow \cdots \Rightarrow C}{C}
\]
Reasoning by Cases, Defeasibly

- strict rules (“→”) vs. defeasible rules (“⇒”)
- schematically:

\[
\begin{array}{c}
A \lor B & A \Rightarrow C & B \Rightarrow C \\
\hline
C
\end{array}
\]

- or, more generally:

\[
\begin{array}{c}
A \lor B & A \Rightarrow \cdots \Rightarrow C & B \Rightarrow \cdots \Rightarrow C \\
\hline
C
\end{array}
\]

- or, more generally:

\[
\begin{array}{c}
A \lor B & A \vdash C & B \vdash C \\
\hline
C
\end{array}
\]

Read \( A \vdash C \): “\( C \) follows defeasibly from \( A \)” or “There is a (defeasible) argument for \( C \) based on \( A \).”
• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]
The Meta-Rule Approach: OR

• Rules for rules:

$$
\frac{A \implies C \quad B \implies C}{A \lor B \implies C} \quad \text{[OR]}
$$

• Illustration:
The Meta-Rule Approach: OR

• Rules for rules:

\[
\begin{align*}
A \Rightarrow C & \quad B \Rightarrow C \\
A \lor B \Rightarrow C
\end{align*}
\]

[OR]

• Illustration:
  1. \( A \Rightarrow C \)
The Meta-Rule Approach: OR

• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]

• Illustration:
  1. \( A \Rightarrow C \)  \( \text{PREM} \)
  2. \( B \Rightarrow C \)  \( \text{PREM} \)
The Meta-Rule Approach: OR

• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad [\text{OR}]
\]

• Illustration:

1. \( A \Rightarrow C \)  
2. \( B \Rightarrow C \)  
3. \( A \lor B \)
The Meta-Rule Approach: OR

• Rules for rules:

\[
\begin{align*}
A \implies C & \quad B \implies C \\
\hline
A \lor B \implies C
\end{align*}
\] [OR]

• Illustration:

1. \( A \implies C \)  
   PREM
2. \( B \implies C \)  
   PREM
3. \( A \lor B \)  
   PREM
4. \( A \lor B \implies C \)  
   1,2; OR
The Meta-Rule Approach: OR

• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]

• Illustration:

1. \( A \Rightarrow C \)  \hspace{1cm} \text{PREM}
2. \( B \Rightarrow C \)  \hspace{1cm} \text{PREM}
3. \( A \lor B \)  \hspace{1cm} \text{PREM}
4. \( A \lor B \Rightarrow C \)  \hspace{1cm} 1,2; OR
5. \( C \)  \hspace{1cm} 3,4; DefeasibleMP
A Problematic Example for OR

Suppose we have
\[ \Sigma = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p \}. \]
A Problematic Example for OR

- By (OR): from $s$ and $u$
- By (Right-Weakening), from $q$ and $r$
- By (OR): from $q_1$ and $u$
A Problematic Example for OR

- by (OR): from $s \Rightarrow v$ and $u \Rightarrow v$
A Problematic Example for OR

- by (OR): from $s \Rightarrow v$ and $u \Rightarrow v$
- by (Right-Weakening), from $q \Rightarrow s$ and $r \Rightarrow u$
A Problematic Example for OR

- by (OR): from $s \Rightarrow v$ and $u \Rightarrow v$
- by (Right-Weakening), from $q \Rightarrow s$ and $r \Rightarrow u$
- by (OR): from $q \Rightarrow s \lor u$ and $r \Rightarrow s \lor u$
A Problematic Example for OR

- Suppose now we also have $t$ and $t \implies \neg s$.
- the possible defeater has no effect on the generalized path
• Suppose now we also have $t'$ and $t' \Rightarrow \neg r$.
• the additional possible defeater has no effect on the generalized path
Extension-based Approaches: Default Logic (Reiter)

- **Input**: set of defaults and a set of formulas ("facts")
- Build **extensions** by applying Modus Ponens to defaults while maintaining consistency
Extension-based Approaches: Default Logic (Reiter)

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- For instance:
Extension-based Approaches: Default Logic (Reiter)

- **Input**: set of defaults and a set of formulas ("facts")
- **Build extensions** by applying Modus Ponens to defaults while maintaining consistency
- For instance:

```
Republican
       ↑
is a

Nixon

is a

Quaker

       ↓

Pacifist

       ↓

¬Pacifist
```

- **Extensions**:
  1. \{Nixon, Republican, Quaker, ¬Pacifist\}
  2. \{Nixon, Republican, Quaker, Pacifist\}
• no handling of disjunctive facts “out-of-the-box”
• for instance: $\Sigma = \{\text{Republican} \lor \text{Democrat}, \text{Republican} \Rightarrow \text{political}, \text{Democrat} \Rightarrow \text{political}\}$.

• since the default is not triggered by the fact, MP cannot be applied
Extension-based Approaches: Default Logic (Reiter)

- idea: split the factual part of the knowledge base (Gelfond, Lifschitz, Przymusinska, 1991)

```
Republican
Base 1

Republican v Democrat
Base 2

Republican

Democrat

Democrat

political
```
Extension-based Approaches: Default Logic (Reiter)

- idea: split the factual part of the knowledge base (Gelfond, Lifschitz, Przymusinska, 1991)

- two extensions:
  1. Republican, political
  2. Democrat, political
Consider the following example:

1. Either his left hand or his right hand is broken. lhb ∨ rhb
Consider the following example:

1. Either his left hand or his right hand is broken. \( \text{lhb} \lor \text{rhb} \)

2. If somebody writes legibly then usually the right hand is not broken. \( \text{wl} \rightarrow \neg \text{rhb} \)
Consider the following example:

1. Either his left hand or his right hand is broken. \( lhb \lor rhb \)
2. If somebody writes legibly then usually the right hand is not broken. \( wl \Rightarrow \neg rhb \)
3. He writes legibly. \( wl \)
Consider the following example:

1. Either his left hand or his right hand is broken. \( lhb \lor rhb \)
2. If somebody writes legibly then usually the right hand is not broken. \( wl \Rightarrow \neg rhb \)
3. He writes legibly. \( wl \)

With disjunctive default logic we get two extensions:

1. \( wl, \neg rhb, lhb \)
2. \( wl, rhb \)
1. Either he’s a left-hander or a right-hander. \((lh \lor rh)\)
2. Usually he’s not a left-hander. \((T \Rightarrow \neg lh)\)
General Stratagem

So far: *Manipulate the database!*

1. produce new defeasible rules from the given ones; or
2. produce new factual knowledge bases when confronted with disjunctive information (by “splitting up” the given knowledge base)
Enters: the Argumentative Approach

- Instead of manipulating the knowledge base and reasoning on top of the manipulated database,
- we will, in what follows, use a more direct approach to the modeling of Reasoning by Cases in the context of defeasible reasoning, following the inference scheme:

\[
A \lor B \quad A \Rightarrow \cdots \Rightarrow C \quad B \Rightarrow \cdots \Rightarrow C
\]

\[
\frac{}{C}
\]

or, more generally:

\[
A \lor B \quad A \not\Rightarrow C \quad B \not\Rightarrow C
\]

\[
\frac{}{C}
\]

- This will allow us to have more control over defeating conditions ...
- ... and to avoid pitfalls as the ones demonstrated above.
Structured Argumentation
The Argumentative Approach to Defeasible Reasoning

- *monotonic logic*: all about **support** and reasons
- *defeasible reasoning*: possibility of **defeat**
- formal argumentation: division of labor
  - rule system: generate arguments
  - argumentation semantics: take care of defeat
The Abstract Dung-Semantics (Dung, 1995)

- abstract away from content of arguments
- argumentation framework: \(\langle \text{Args}, \text{Attacks} \rangle\) (directed graph)
The Abstract Dung-Semantics i (Dung, 1995)

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- requirements, e.g.,
  - conflict-free
  - admissible (each argument can be defended)
• abstract away from content of arguments
• argumentation framework: ⟨Args, Attacks⟩ (directed graph)
• determine selections of arguments that represent stances of rational reasoners in the respective debate
• requirements, e.g.,
  • conflict-free
  • admissible (each argument can be defended)
• First select unattacked arguments
• Remove the arguments attacked by the selected arguments
• Select unattacked arguments
• Remove the arguments attacked by the selected arguments
• And so on … until fixed point is reached
• first select unattacked arguments
• first select unattacked arguments
• remove the arguments attacked by the selected arguments
Grounded Semantics

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Grounded Semantics

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- and so on ... until fixed point is reached
Away with the abstraction! Structured Argumentation ...
The landscape of structured argumentation

1. Deductive argumentation
   1.1 no defeasible rules, no defeasible premises
   1.2 semi-abstract: arguments have form \((\Gamma, A)\) where \(\Gamma \vdash A\) for appropriate \(\vdash\)
   1.3 e.g.: Besnard & Hunter, Arieli & Straßer

2. Assumption-based argumentation (Dung, Kowalski, Toni)
   2.1 no defeasible rules but defeasible premises
   2.2 semi-abstract: arguments are sets of defeasible premises

3. Rule/Proof-based argumentation
   3.1 strict and defeasible rules
   3.2 non-abstract: arguments as proofs
   3.3 ASPIC (Prakken, Modgil, …)
... consist of

• a set $S$ of strict rules of the form $\phi_1, \ldots, \phi_n \rightarrow \psi$ (in this talk: $\phi_1, \ldots, \phi_n \rightarrow \psi$ iff $\phi_1, \ldots, \phi_n \vdash_{\text{CL}} \psi$)
Argumentation Theories (simplified)

... consist of

- a set $S$ of strict rules of the form $\phi_1, \ldots, \phi_n \rightarrow \psi$ (in this talk: $\phi_1, \ldots, \phi_n \rightarrow \psi$ iff $\phi_1, \ldots, \phi_n \vdash_{\text{CL}} \psi$)
- a set $D$ of defeasible rules of the form $\phi_1, \ldots, \phi_n \Rightarrow \psi$
... consist of

- a set $S$ of strict rules of the form $\phi_1, \ldots, \phi_n \rightarrow \psi$ (in this talk: $\phi_1, \ldots, \phi_n \rightarrow \psi$ iff $\phi_1, \ldots, \phi_n \vdash_{\text{CL}} \psi$)
- a set $D$ of defeasible rules of the form $\phi_1, \ldots, \phi_n \Rightarrow \psi$
- a set $K$ of premises (“facts”) (formulas without occurrences of $\rightarrow$ and $\Rightarrow$)
Let the argumentation theory $T = \langle S, D, K \rangle$ consist of:

- $S = \{\phi_1, \ldots, \phi_n \rightarrow \psi \mid \phi_1, \ldots, \phi_n \vdash_{CL} \psi\}$
- $D = \{p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s\}$
- $K = \{p, t\}$. 
Idea: Arguments as proof trees.
In a nutshell: $T$-arguments are proof trees based on $T = \langle S, D, K \rangle$. 
Arguments provided by an argumentation theory

In a nutshell: $T$-arguments are proof trees based on $T = \langle S, D, K \rangle$.

In detail:

- for every $\phi \in K$, $A = \langle \phi \rangle$ is an argument with conclusion $\text{Conc}(A) = \phi$ and the only subargument $\text{Sub}(A) = \{\phi\}$
- for all arguments $A_1, \ldots, A_n$,
  - if $\text{Con}(A_1), \ldots, \text{Con}(A_n) \rightarrow \psi \in S$ then $A = \langle A_1, \ldots, A_n \rightarrow \psi \rangle$ is an argument for which $\text{Conc}(A) = \psi$ and $\text{Sub}(A) = \{A\} \cup \text{Sub}(A_1) \cup \cdots \cup \text{Sub}(A_n)$
  - if $\text{Con}(A_1), \ldots, \text{Con}(A_n) \Rightarrow \psi \in D$ then $A = \langle A_1, \ldots, A_n \Rightarrow \psi \rangle$ is an argument for which $\text{Conc}(A) = \psi$ and $\text{Sub}(A) = \{A\} \cup \text{Sub}(A_1) \cup \cdots \cup \text{Sub}(A_n)$
Arguments provided by an argumentation theory

In a nutshell: $T$-arguments are proof trees based on $T = \langle S, D, K \rangle$.

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  - if $\text{Con}(A_1), \ldots, \text{Con}(A_n) \Rightarrow \psi \in D$ then $A = \langle A_1, \ldots, A_n \Rightarrow \psi \rangle$ is an argument for which $\text{Con}(A) = \psi$ and $\text{Sub}(A) = \{A\} \cup \text{Sub}(A_1) \cup \cdots \cup \text{Sub}(A_n)$

We write $\text{Arg}(T)$ for the set of all arguments based on $T$. 
The dialectic tier of argumentation: attacks!
Argumentative Attacks

ASPIC provides several attack forms such as rebuts and undercuts. Here we restrict ourselves to the former.

Where $A, B \in \text{Arg}(T)$, a rebuts $B = \langle B_1, \ldots, B_n \Rightarrow \psi \rangle$ if

- $\text{Con}(A) = \neg \psi$ or $\psi = \neg \text{Conc}(A)$ or
- there is a $B' \in \text{Sub}(B)$ such that $A$ rebuts $B'$.
• The **argumentation framework** $AF$ based on $T$ is given by $\langle \text{Arg}(T), \text{Attack}(T) \rangle$. 
• The argumentation framework $AF$ based on $T$ is given by $\langle \text{Arg}(T), \text{Attack}(T) \rangle$.

• This is used to define a consequence relation, e.g., as follows:

$$T \vdash \phi \text{ iff there is a grounded argument } A \in \text{Arg}(T) \text{ with } \text{Conc}(A) = \phi.$$
Time for an example!
\[\mathcal{D} = \{p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s\}\]
\[\mathcal{K} = \{p, t, q\}\]
\[ \mathcal{D} = \{ p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \} \]

\[ \mathcal{K} = \{ p, t, q \}. \]

We have, e.g., the arguments:

- \( A_1 = \langle q \rangle, A_2 = \langle r \rangle, A'_3 = \langle t \rangle \)
- \( A_3 = \langle \langle t \rangle \Rightarrow \neg s \rangle \)
- \( A_4 = \langle \langle q \rangle \Rightarrow s \rangle \)
- \( A_5 = \langle A_4 \Rightarrow v \rangle \)
- \( A_6 = \langle \langle r \rangle \Rightarrow u \rangle \)
- \( A_7 = \langle A_6 \Rightarrow v \rangle \)

Abstract perspective:
\[ \mathcal{D} = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \} \]

\[ \mathcal{K} = \{ p, t, q \}. \]

We have, e.g., the arguments:

- \( A_1 = \langle q \rangle \), \( A_2 = \langle r \rangle \), \( A_3 = \langle t \rangle \)
- \( A_3 = \langle \langle t \rangle \Rightarrow \neg s \rangle \)
- \( A_4 = \langle \langle q \rangle \Rightarrow s \rangle \)
- \( A_5 = \langle A_4 \Rightarrow v \rangle \)
- \( A_6 = \langle \langle r \rangle \Rightarrow u \rangle \)
- \( A_7 = \langle A_6 \Rightarrow v \rangle \)

More structured perspective:

- \( A_3 : \)
  \[ \begin{array}{cc}
    t & \rightarrow & \neg s \\
    \end{array} \]

- \( A_5 : \)
  \[ \begin{array}{ccc}
    q & \rightarrow & s & \rightarrow & v \\
    \end{array} \]

- \( A_7 : \)
  \[ \begin{array}{ccc}
    r & \rightarrow & u & \rightarrow & v \\
    \end{array} \]
Intermezzo: contamination and its cures.
Intermezzo: The Contaminating Power of Inconsistent Arguments

- Suppose $\mathcal{D} = \{ \implies p, \implies \neg p, \implies s \}$ and $\mathcal{K} = \emptyset$
Intermezzo: The Contaminating Power of Inconsistent Arguments

- Suppose $\mathcal{D} = \{ \rightarrow p, \rightarrow \neg p, \rightarrow s \}$ and $\mathcal{K} = \emptyset$
- In this case we clearly want $s$ to be a consequence.
Intermezzo: The Contaminating Power of Inconsistent Arguments

• Suppose $\mathcal{D} = \{ \Rightarrow p, \Rightarrow \neg p, \Rightarrow s \}$ and $\mathcal{K} = \emptyset$

• In this case we clearly want $s$ to be a consequence.

• Let’s see what happens. We have, for instance, the arguments:
Intermezzo: The Contaminating Power of Inconsistent Arguments

• Suppose $\mathcal{D} = \{ \Rightarrow p, \Rightarrow \neg p, \Rightarrow s \}$ and $\mathcal{K} = \emptyset$

• In this case we clearly want $s$ to be a consequence.

• Let’s see what happens. We have, for instance, the arguments:

  • $a_p = \langle \Rightarrow p \rangle$
  • $a_{\neg p} = \langle \Rightarrow \neg p \rangle$
  • $a_s = \langle \Rightarrow s \rangle$
  • $a_\bot = \langle a_p, a_{\neg p} \rightarrow \neg s \rangle$
Solutions to this problem

- Filter out inconsistent arguments (Wu, PhD, 2012)
- “Fact-attack” inconsistent arguments (here)
- Generalized Rebut (Jesse’s talk)
- When is an argument inconsistent?
Solutions to this problem

- Filter out inconsistent arguments (Wu, PhD, 2012)
- “Fact-attack” inconsistent arguments (here)
- Generalized Rebut (Jesse’s talk)
- When is an argument inconsistent?

**Definition**
Where $A \in \text{Arg}(T)$ we define $\dagger(A)$ as inductively follows:

- $\dagger(\langle \phi \rangle) = \phi$
- $\dagger(\langle A_1, \ldots, A_n \rightarrow \phi \rangle) = \dagger(A_1) \land \cdots \land \dagger(A_n)$
- $\dagger(\langle A_1, \ldots, A_n \Rightarrow \phi \rangle) = \dagger(A_1) \land \cdots \land \dagger(A_n) \land \phi$

**Definition**
An argument $A \in \text{Arg}(T)$ is inconsistent iff $\mathcal{K} \vdash \neg \dagger(A)$. 

Definition
Where $A \in \text{Arg}(T)$ we define $\tau(A)$ as inductively follows:

\[
\begin{align*}
\tau(\langle \phi \rangle) &= \phi \\
\tau(\langle A_1, \ldots, A_n \rightarrow \phi \rangle) &= \tau(A_1) \land \cdots \land \tau(A_n) \\
\tau(\langle A_1, \ldots, A_n \Rightarrow \phi \rangle) &= \tau(A_1) \land \cdots \land \tau(A_n) \land \phi
\end{align*}
\]

Definition
An argument $A \in \text{Arg}(T)$ is **inconsistent** iff $\mathcal{K} \vdash \neg \tau(A)$.

Definition
A strict argument\(^1\) $A \in \text{Arg}(T)$ **fact-attacks** some argument $B \in \text{Arg}(T)$ iff $\text{Conc}(A) = \neg \tau(B)$.

---

\(^1\)An argument is strict if it doesn’t have any defeasible rules.
Reasoning by Cases in Structured Argumentation
Basic idea: Given

• an argumentation theory $T = \langle D, K \rangle$,  

\footnote{\textsuperscript{2}Since in this talk the strict rules $S$ will always be provided by classical logic, we will not mention them anymore.}
A New Type of Argument: RbC-Arguments

Basic idea: Given

- an argumentation theory $T = \langle \mathcal{D}, \mathcal{K} \rangle$,\(^2\)
- an argument $A \in \text{Arg}(T)$ for which $\text{Conc}(A) = \phi_1 \lor \phi_2$,

\(^2\)Since in this talk the strict rules $\mathcal{S}$ will always be provided by classical logic, we will not mention them anymore.
Basic idea: Given

- an argumentation theory $T = \langle D, K \rangle$,\(^2\)
- an argument $A \in \text{Arg}(T)$ for which $\text{Conc}(A) = \phi_1 \lor \phi_2$,
- an argument $A_1[\phi_1] \in \text{Arg}(\langle D, K \rangle)$ with $\text{Conc}(A_1) = \psi$, and

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Basic idea: Given

- an argumentation theory $T = \langle \mathcal{D}, \mathcal{K} \rangle$, \(^2\)
- an argument $A \in \text{Arg}(T)$ for which $\text{Conc}(A) = \phi_1 \lor \phi_2$,
- an argument $A_1[\phi_1] \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \rangle)$ with $\text{Conc}(A_1) = \psi$, and
- an argument $A_2[\phi_2] \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \rangle)$ with $\text{Conc}(A_2) = \psi$,

---

\(^2\)Since in this talk the strict rules $S$ will always be provided by classical logic, we will not mention them any more.
Basic idea: Given

- an argumentation theory $T = \langle D, K \rangle$, \(^2\)
- an argument $A \in \text{Arg}(T)$ for which $\text{Conc}(A) = \phi_1 \lor \phi_2$,
- an argument $A_1[\phi_1] \in \text{Arg}(\langle D, K \rangle)$ with $\text{Conc}(A_1) = \psi$, and
- an argument $A_2[\phi_2] \in \text{Arg}(\langle D, K \rangle)$ with $\text{Conc}(A_2) = \psi$,

we introduce a new RbC-Argument

$$\langle A, A_1[\phi_1], A_2[\phi_2] \rightsquigarrow \psi \rangle$$

\(^2\)Since in this talk the strict rules $\mathcal{S}$ will always be provided by classical logic, we will not mention them anymore.
The big picture ...
More general: RbC-Argument

Definition (Fact-Introduction)
\[ A[\emptyset] = \langle \varphi \rangle \text{ where } \varphi \in \mathcal{F} \]
- \( \text{Con}(A[\emptyset]) = \varphi \)
- \( \text{Sub}(A[\emptyset]) = \{A[\emptyset]\} \)
- \( \text{HSub}(A[\emptyset]) = \emptyset \)

Definition (Hypothesis-Introduction)
\[ A[\{\varphi\}] = [\varphi] \text{ where } \varphi \in \mathcal{L} \]
- \( \text{Con}(A[\{\varphi\}]) = \varphi \)
- \( \text{Sub}(A[\{\varphi\}]) = \{A[\{\varphi\}]\} \)
- \( \text{HSub}(A[\{\varphi\}]) = \emptyset \)
Definition (Arguments with deductive top-rule)

\[ A[\Theta] = A_1[\Delta_1], \ldots, A_n[\Delta_n] \rightarrow \varphi \]

where \( A_1[\Delta_1], \ldots, A_n[\Delta_n] \in \text{Arg}(AT) \), \( \text{Con}(A_1[\Delta_1]), \ldots, \text{Con}(A_n[\Delta_n]) \rightarrow \varphi \in S \), and \( \Theta = \Delta_1 \cup \ldots \cup \Delta_n \)

- \( \text{Con}(A[\Theta]) = \varphi \)
- \( \text{Sub}(A[\Theta]) = \{A[\Theta]\} \cup \text{Sub}(A_1[\Delta_1]) \cup \ldots \cup \text{Sub}(A_n[\Delta_n]) \)
- \( \text{HSub}(A[\Theta]) = \text{HSub}(A_1[\Delta_1]) \cup \ldots \cup \text{HSub}(A_n[\Delta_n]) \)
Definition (Arguments with defeasible top-rule)

\[ A[\Theta] = A_1[\Delta_1], \ldots, A_n[\Delta_n] \Rightarrow \varphi \]

where \( A_1[\Delta_1], \ldots, A_n[\Delta_n] \in \text{Arg}(AT) \),
\( \text{Con}(A_1[\Delta_1]), \ldots, \text{Con}(A_n[\Delta_n]) \Rightarrow \varphi \in \mathcal{D}, \Theta = \Delta_1 \cup \ldots \cup \Delta_n \),
and \( \text{HSub}(A_i[\Delta_i]) = \emptyset \) for every \( 1 \leq i \leq n \).

- \( \text{Con}(A[\Theta]) = \varphi \)
- \( \text{Sub}(A[\Theta]) = \{A[\Theta]\} \cup \text{Sub}(A_1[\Delta_1]) \cup \ldots \cup \text{Sub}(A_n[\Delta_n]) \)
- \( \text{HSub}(A[\Theta]) = \text{HSub}(A_1[\Delta_1]) \cup \ldots \cup \text{HSub}(A_n[\Delta_n]) \)
Definition (RbC-Arguments)

\[ A[\Theta] = A_1[\Theta], A_2[\Delta_2], \ldots, A_n[\Delta_n] \sim \varphi \]

where \( n \geq 2 \), \( \varphi = \bigvee \{ \text{Con}(A_2[\Delta_2]), \ldots, \text{Con}(A_n[\Delta_n]) \} \), \( \text{HSub}(A_1[\Theta]) = \emptyset \), \( \text{Con}(A_1[\Delta_1]) = \bigvee_{i=2}^{n} \psi_i \), and for all \( i \in \{2, \ldots, n\} \): \( \psi_i \in \Delta_i \) and \( \Delta_i \subseteq \Theta \cup \{\psi_i\} \). We have:

- \( \text{Con}(A[\Theta]) = \varphi \)
- \( \text{Sub}(A[\Theta]) = \{A[\Theta]\} \cup \text{Sub}(A_1[\Delta_1]) \)
- \( \text{HSub}(A[\Theta]) = \{A_2[\Delta_2], \ldots, A_n[\Delta_n]\} \)
Let $T = \langle \mathcal{D}, \mathcal{K} \rangle$ consist of
\[
\mathcal{D} = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \} \text{ and } \mathcal{K} = \{ p, t \}.
\]
Let $T = \langle D, K \rangle$ consist of

$D = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \}$ and $K = \{ p, t \}$.

We have for instance the arguments:

- $A_1[\emptyset] = \langle \langle p \rangle \Rightarrow q \lor r \rangle$
- $A_2[q] = \langle [q] \Rightarrow s \Rightarrow v \rangle$
- $A_3[r] = \langle [r] \Rightarrow u \Rightarrow v \rangle$
- $A_4[\emptyset] = \langle A_1[\emptyset], A_2[q], A_3[r] \Rightarrow v \rangle$. 
What about attacks?
\[
\begin{align*}
A_1[\varnothing] &= \langle \langle p \rangle \Rightarrow q \lor r \rangle \\
A_2[q] &= \langle [q] \Rightarrow s \Rightarrow v \rangle \\
A_3[r] &= \langle [r] \Rightarrow u \Rightarrow v \rangle \\
A_4[\varnothing] &= \langle A_1[\varnothing], A_2[q], A_3[r] \Rightarrow v \rangle.
\end{align*}
\]
• $A_1[\emptyset] = \langle\langle p \Rightarrow q \lor r \rangle\rangle$
• $A_2[q] = \langle[q] \Rightarrow s \Rightarrow v\rangle$
• $A_3[r] = \langle[r] \Rightarrow u \Rightarrow v\rangle$
• $A_4[\emptyset] = \langle A_1[\emptyset], A_2[q], A_3[r] \Rightarrow v \rangle$.
• $A_5[t] = \langle[t] \Rightarrow \neg s\rangle$
\[ A_1[\emptyset] = \langle\langle p \Rightarrow q \vee r \rangle \rangle \]
\[ A_2[q] = \langle\langle [q] \Rightarrow s \Rightarrow v \rangle \rangle \]
\[ A_3[r] = \langle\langle [r] \Rightarrow u \Rightarrow v \rangle \rangle \]
\[ A_4[\emptyset] = \langle A_1[\emptyset], A_2[q], A_3[r] \Rightarrow v \rangle. \]
• \(A_1[\emptyset] = \langle\langle p \Rightarrow q \lor r \rangle\rangle\)
• \(A_2[q] = \langle[q] \Rightarrow s \Rightarrow v \rangle\)
• \(A_3[r] = \langle[r] \Rightarrow u \Rightarrow v \rangle\)
• \(A_4[\emptyset] = \langle A_1[\emptyset], A_2[q], A_3[r] \Rightarrow v \rangle\).
• \(A_6[q] = \langle\langle q \Rightarrow \neg s \rangle\rangle\).
Definition (Direct Rebut)
Where $A[\Theta], B[\Delta] \in \text{Arg}(T)$, $A[\Theta]$ directly rebuts $B[\Delta]$ iff

- $\text{Con}(A) = \neg \text{Con}(B)$ or $\neg \text{Con}(A) = \text{Con}(B)$, and
- $\Delta \vdash \land \Theta$.

Definition (Rebut)
Intermezzo: the other approach to rbc-defeat.
We could demand that an attacker shows that each hypothetical subargument is problematic. For instance,

\[
\begin{align*}
C_1: & \quad r & \rightarrow & \neg p \\
C: & \quad r \lor \neg r & & \\
C_2: & \quad \neg r & \rightarrow & \neg q \\
A: & \quad T & \rightarrow & p \land q
\end{align*}
\]
But, this leads to trouble. Suppose we have \{ T \Rightarrow \neg s; \neg p \Rightarrow s \} then we can infer \( p \). So we can contrapose defeasible conditionals. Here is how:

- \( A = T \Rightarrow \neg s \)
- \( B = p \lor \neg p, [p], [\neg p \Rightarrow s] \sim p \lor s \)
- \( C = A, B \rightarrow p \)

**Modus Tollens** is often considered as not desirable for nonmonotonic conditionals. We’ll go with the mainstream (a bit longer).
Suppose $D = \{ T \Rightarrow s \}$ and $\mathcal{F} = \emptyset$. Then under any semantics $s$ is derivable.

Suppose we have $D_2 = D \cup \{ T \Rightarrow p; \ r \Rightarrow \neg p; \ T \Rightarrow r \}$.

- $A = T \Rightarrow s$
- $B = T \Rightarrow p$
- $C = T \Rightarrow r$
- $D = \langle C \rightarrow r \lor \neg s \rangle, [r] \Rightarrow \neg p, [-s] \rightsquigarrow \neg p \lor \neg s$
- $E = D, B \rightarrow \neg s$
- $F = A, B \rightarrow s \land p$

**Problem:** now $s$ doesn’t follow in most semantics.
Our approach is not without difficulties either: a pragmatic oddity.
• If it’s a working day she usually works. $wd \Rightarrow w$
• If it’s a working day and there is a traffic jam, she is usually not in the library. $wd \land tj \Rightarrow \neg l$
• If she works but is not in the library she is usually in a cafe. $w \land \neg l \Rightarrow c$
• If she works in a cafe she usually types her thesis. $w \land c \Rightarrow t$
• If it’s a working day and there is no traffic jam, she usually works in the library. $wd \land \neg tf \Rightarrow l$
• If she works in the library she usually reads. $l \Rightarrow r$
• She’s not in the library. $\neg l$
Generalizing Fact-attacks ...
Fact-attacks again

Definition
Where $A \in \text{Arg}(T)$ we define $\dagger(A)$ as inductively follows:

- $\dagger(\langle \phi \rangle) = \phi$
- $\dagger(\langle A_1, \ldots, A_n \rightarrow \phi \rangle) = \dagger(A_1) \land \cdots \land \dagger(A_n)$
- $\dagger(\langle A_1, \ldots, A_n \Rightarrow \phi \rangle) = \dagger(A_1) \land \cdots \land \dagger(A_n) \land \phi$
- $\dagger(\langle A_1, [A_2], \ldots, [A_n] \rightsquigarrow \phi \rangle) = \dagger(A_1) \land (\dagger(A_2) \lor \cdots \lor \dagger(A_n))$

Definition
A strict argument $A \in \text{Arg}(T)$ fact-attacks some argument $B \in \text{Arg}(T)$ iff $\text{Con}(A) = \neg \dagger(B)$. 
Suppose $\mathcal{D} = \{T \Rightarrow s\}$ and $\mathcal{F} = \emptyset$. Then under any semantics $s$ is derivable.
Suppose $\mathcal{D} = \{ T \Rightarrow s \}$ and $\mathcal{F} = \emptyset$. Then under any semantics $s$ is derivable.

Suppose we have $\mathcal{D}_2 = \mathcal{D} \cup \{ T \Rightarrow p; \ r \Rightarrow \neg p; \ T \Rightarrow r \}$. 
Contamination again

Suppose $\mathcal{D} = \{T \Rightarrow s\}$ and $\mathcal{F} = \emptyset$. Then under any semantics $s$ is derivable.

Suppose we have $\mathcal{D}_2 = \mathcal{D} \cup \{T \Rightarrow p; \ r \Rightarrow \neg p; \ T \Rightarrow r\}$.

- $A = T \Rightarrow s$
- $B = T \Rightarrow p$
- $C = T \Rightarrow r$
- $C' = C \Rightarrow \neg p$
- $D = \langle C \rightarrow r \lor \neg s \rangle, [r] \Rightarrow \neg p, [\neg s] \rightsquigarrow \neg p \lor \neg s$
- $E = D, B \rightarrow \neg s$

Problem: now $s$ doesn’t follow in grounded semantics.
Idea $A$ is consistent iff for each h-path $(\delta_1, \ldots, \delta_n)$ of $A$, 
\[ \bigwedge_{i=1}^{n} \delta_i \land \vdash A \models \bot. \]
Idea $A$ is consistent iff for each h-path $(\delta_1, \ldots, \delta_n)$ of $A$, 
$\bigwedge_{i=1}^{n} \delta_i \land \top A \not\vdash \bot$. 

h-paths: Illustration instead of a definition
Idea $A$ is consistent iff for each h-path $(\delta_1, \ldots, \delta_n)$ of $A$, 
$\bigwedge_{i=1}^{n} \delta_i \land \vdash A \not\vdash \bot.$
h-paths: Illustration instead of a definition

Idea $A$ is consistent iff for each h-path $(\delta_1, \ldots, \delta_n)$ of $A$, $\bigwedge_{i=1}^{n} \delta_i \land \vdash A \Downarrow \bot$. 
Idea $A$ is consistent iff for each h-path $(\delta_1, \ldots, \delta_n)$ of $A$, 
$\bigwedge_{i=1}^{n} \delta_i \land \vdash A \vdash \bot$. 

$h$-paths: Illustration instead of a definition
How does this help with the example?

- $A = T \implies s$
- $B = T \implies p$
- $C = T \implies r$
- $C' = C \implies \neg p$
- $D = \langle C \to r \lor \neg s \rangle, [r] \implies \neg p, [\neg s] \rightsquigarrow \neg p \lor \neg s$
- $E = D, B \to \neg s$

Note that $\vdash (E) = r \land ((r \land \neg p) \lor \neg s) \land p$.

One h-path is via $[r] \implies \neg p$. 
Consequences

We define a consequence relation on basis of $AT = \langle \text{Arg}(T), \text{Attacks}(T) \rangle$ as before on the basis of the grounded arguments:

$$AT \models \phi \text{ iff there is a grounded argument } A[\emptyset] \in \text{Arg}(T) \text{ for which } \text{Con}(A) = \phi.$$
Let $T$ be such that $\mathcal{D} = \{\text{wl} \Rightarrow \neg \text{rhb}\}$ and $\mathcal{K} = \{\text{wl}, \text{lhb} \lor \text{rhb}\}$.

We have the following arguments:

- $a_1 = \langle \text{lhb} \lor \text{rhb} \rangle$
- $a_2 = \langle \langle \text{wl} \rangle \Rightarrow \neg \text{rhb} \rangle$
- $a_3 = \langle a_1, a_2 \rightarrow \text{lh} \rangle$.

So lhb is derivable, as expected.
Complications
Closure
No Closure

Problem: \( C = \langle A, B \rightarrow t \land u \rangle \) is fact-attacked while \( A \) and \( B \) are grounded. Therefore: no closure!
Underdetermined attacks

Problem: It seems that $A$ and $C$ form an inconsistent stance.
Idea: Generalize rebuts to disjunctive rebuts
Definition
An rbc-tree in $B$ is a labeled rooted tree where each label is either an rbc-argument or a non-rbc non-strict argument.

- The root of $\tau$ is $B$.
- Each non-rbc node $A$ is either a leaf or it has an rbc-node $C \in \text{Sub}(A)$ as only child.
- For each rbc-node $\langle A_0, A_1, \ldots, A_n \leadsto \phi \rangle$ and each $1 \leq i \leq n$ there is maximal one child node $A_i$. 
Solution: Disjunctive Attacks

Definition
An rbc-tree in $B$ is a labeled rooted tree where each label is either an rbc-argument or a non-rbc non-strict argument.

• The root of $\tau$ is $B$.
• Each non-rbc node $A$ is either a leaf or it has an rbc-node $C \in \text{Sub}(A)$ as only child.
• For each rbc-node $\langle A_0, A_1, \ldots, A_n \leadsto \phi \rangle$ and each $1 \leq i \leq n$ there is maximal one child node $A_i$.

Definition
$A[\Delta]$ disjunctively rebuts $B[\Delta']$ if $\text{Con}(A[\Delta]) = \bigvee \bar{\phi_i}$ and there is an rbc-tree in $B$ with leaves $B_1, \ldots, B_n$ such that for each $1 \leq i \leq n$ there is a $B' = \langle \ldots \Rightarrow \phi_i \rangle \in \text{Sub}(B_i)$ and $\Delta' \vdash F \land \Delta$. 
Some rbc-trees of $A$: 

- $B$: 
  - $B_1$: $s \rightarrow p \rightarrow p_1 \rightarrow u$ 
  - $B_2$: $q \rightarrow p_2 \rightarrow u$ 

- $C$: 
  - $C_1$: $s' \rightarrow p' \rightarrow p'_1 \rightarrow u'$ 
  - $C_2$: $q' \rightarrow p'_2 \rightarrow u'$ 

- $A$: 
  - $s \lor s'$
$D[\emptyset] = \langle \cdots \Rightarrow \neg p_1 \lor \neg p_1' \rangle$

disjunctively rebuts $A[\emptyset]$ relative to $\tau_1$. 
Back to Underdetermined Attacks

Now: $C$ disjunctively attacks $A$ relative to $A_1$ and $A_2$. 
Back to No Closure

A: $p \lor q$

B: $r \lor s$

$A'$: $p \lor q$

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... but ...
Do Hypotheses have a special attack status?

What if not?

\[ r \lor \neg r \]

\[ \neg r \lor \neg \neg r \]
... but (cont.) ...
Back to the pragmatic oddity
An Impasse?

Problem: C does not disjunctively rebut $D = \langle A, B \rightarrow t \land w \rangle$ according to our definition.

Note: rbc-trees for $D$ are of the form:

$$D$$

$$\begin{array}{c}
\text{D} \\
\text{A} \\
\text{A}_1 \quad \text{A}_2
\end{array}$$

$$\begin{array}{c}
\text{D} \\
\text{B} \\
\text{B}_1 \quad \text{B}_2
\end{array}$$
Other Approach: Generalized Rebut relative to h-paths

Idea: $A$ attacks $B$ iff there is an h-path $(\delta_1, \ldots, \delta_n)$ in $B$ such that $\text{Con}(A) = \neg \wedge \Theta$ where $\Theta \subseteq \{\delta_1, \ldots, \delta_n\}$. 
Other Approach: Generalized Rebut relative to h-paths

Definition

Where $A[\Delta]$ and $B[\Theta]$ are arguments in $\text{Arg}(AT)$, $A[\Delta]$ attacks $B[\Theta]$ in $\langle A_1, \ldots, A_n \rangle$ iff there is an h-path $\langle H_1[\Pi_1], \ldots, H_n[\Pi_n] \rangle$ in $B[\Theta]$ such that each of the following conditions is fulfilled:

1. for each $1 \leq i \leq n$, $A_i \subseteq \text{Sub}(H_i[\Pi_i])$,
2. $\text{Conc}(A[\Delta]) = \neg \bigwedge \bigcup_{i=1}^{n} \text{Concs}(A_i)$,
3. $A_n \neq \emptyset$,
4. at least one $H_i[\Pi_i]$ is defeasible, and
5. $\Pi_i \vdash_{\mathcal{F}} \Delta$ where $i$ is minimal such that $A_i \neq \emptyset$ (where $\mathcal{F}$ is the set of the given facts).
Other Approach: Generalized Rebut relative to h-paths

\[ A : \quad p \lor q \]

\[ B : \quad r \lor s \]

\[ A' : \quad p \lor q \]

\[ \neg(p \land q) \]
... but ... hypothetical closure ...
\[ D = \{ s \Rightarrow p; \ s \Rightarrow q; \ r \Rightarrow \neg p; \ \neg r \Rightarrow \neg q \} \]

and the empty set of facts \( F = \emptyset \). Some arguments:

- \( A[s] = [s] \Rightarrow p \)
- \( B[s] = [s] \Rightarrow q \)
- \( C_1[r] = [r] \Rightarrow \neg p \)
- \( C_2[\neg r] = [\neg r] \Rightarrow \neg q \)
- \( C[\emptyset] = \langle \rightarrow r \lor \neg r \rangle, C_1, C_2 \leadsto \neg p \lor \neg q \rightarrow \neg (p \land q) \)
- \( (A \oplus B)[\emptyset] = A, B \rightarrow p \land q \)

We have the following attacks:

- \( (A \oplus B)[s] \) is attacked by \( C[\emptyset] \) (with unrestricted generalized rebut);
- \( (A \oplus C)[s] \) and \( (B \oplus C)[s] \) are inconsistent and thus filtered out.

This means that we get \( A[s] \) and \( B[s] \) being grounded, while \( (A \oplus B)[s] \) is not grounded.
Trouble with OR for the consequence relation:

If \( \langle S, D, K \cup \{\phi_1}\rangle \models \psi \) and \( \langle S, D, K \cup \{\phi_2}\rangle \models \psi \),
then \( \langle S, D, K \cup \{\phi_1 \lor \phi_2\}\rangle \models \psi \).
OR for the Consequence Relation

\[ A[\emptyset] : p \lor q \]

\[ B[h] : h \quad h' \quad s \]

\[ C[p] : p \quad \neg h'' \]

\[ D[h] : h \quad h'' \quad \neg h' \]

So: \( p \lor q \models s \).
OR for the Consequence Relation (cont.)

A[∅]: $p \lor q$

B[h]: $h \lor f \rightarrow s$

C[∅]: $p \rightarrow \neg h''$

D[h]: $h \rightarrow h'' \rightarrow \neg h'$

So: $p \not|\not s$. 
OR for the Consequence Relation (cont.)

So: \( q \not\models s \).
Idea: all treat last hypotheses used in the originally attacked argument as the weakest in an attack-defense sequence.
Another hard problem
Weak Contraposition
Another hard problem (cont.): Contraposition of defeasible conditionals

C₁: \( r \lor \neg r \)

C₂: \( \neg r \rightarrow \neg q \)

A: \( T \rightarrow p \rightarrow \neg r \rightarrow \neg q \)

B: \( T \rightarrow q \rightarrow r \rightarrow \neg p \)
For reasoning with contrapositions of defeasible conditional one needs reasoning by cases!
Suppose we have $\mathcal{D} = \{s' \Rightarrow s; \ t' \Rightarrow t; \ s, t \Rightarrow p\}$ and $\mathcal{F} = \{\neg p\}$. 
Idea: Distinguish different commitments levels for propositions: qualify propositions.

- We let $\circ \phi$ stand for “I wouldn’t be surprised by $\phi$” / “$\phi$ would be consistent with my belief state” / etc.
- E.g., in terms of likelihood thresholds:

<table>
<thead>
<tr>
<th>Status: $\neg p$</th>
<th>$\neg \circ p$</th>
<th>$\circ \neg p$</th>
<th>$\circ p$</th>
<th>$\neg \circ \neg p$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood of $p$:</td>
<td>$&lt; 20$</td>
<td>$&lt; 40$</td>
<td>$&lt; 60$</td>
<td>$\geq 40$</td>
<td>$\geq 60$</td>
</tr>
</tbody>
</table>
More Subtle Approach

Weak Modus Tollens: if $\phi \Rightarrow \psi \in \mathcal{D}$ then we have the argument $\langle \diamond \neg \psi \Rightarrow \diamond \neg \phi \rangle$.

(More general: if $\phi_1, \ldots, \phi_n \Rightarrow \psi \in \mathcal{D}$ then we have the argument $\langle \diamond \neg \phi \Rightarrow \diamond \neg \bigwedge_{i=1}^{n} \phi_i \rangle$)

Rationale: Suppose

- I consider $\phi \Rightarrow \psi$ to be applicable to the current situation (e.g., I don’t commit to an undercut of it) and
- I consider $\neg \psi$ to be consistent with my belief state / I wouldn’t be surprised by $\neg \psi$,
- then I should consider $\neg \phi$ to be consistent with my belief state / I shouldn’t be surprised by $\neg \phi$.

(What about: $\neg \diamond \psi \Rightarrow \neg \diamond \phi$?)
More Subtle Approach

**Weak Modus Ponens** if $\phi \implies \psi \in \mathcal{D}$ then we have the argument $\langle o\phi \implies o\psi \rangle$.

(More general: if $\phi_1, \ldots, \phi_n \implies \psi \in \mathcal{D}$ then we have the argument $\langle o \bigwedge_{i=1}^{n} \phi_i \implies o\psi \rangle$)

**Rationale:** Suppose

- I consider $\phi \implies \psi$ to be applicable to the current situation (e.g., I don’t commit to an undercut of it) and
- I consider $\phi$ to be consistent with my belief state,
- then I should consider $\psi$ to be consistent with my belief state.
In our example ...

\[ C_1 : r \lor \neg r \]

\[ C_2 : \neg r \lor \neg q \]

\[ A' : \top \rightarrow p \rightarrow \neg \neg p \rightarrow \neg r \rightarrow \neg q \]

\[ B' : \top \rightarrow q \rightarrow \neg \neg q \rightarrow \neg r \rightarrow \neg p \]
Back to the problem with closure

A: \( p \lor q \)

B: \( r \lor s \)

A': \( p \lor q \)

Diagram:

- \( p \rightarrow p_1 \rightarrow p_2 \rightarrow t \)
- \( q \rightarrow q_1 \rightarrow q_2 \rightarrow t \)
- \( r \rightarrow \neg p_1 \rightarrow \neg q_1 \rightarrow u \)
- \( s \rightarrow \neg p_2 \rightarrow \neg q_2 \rightarrow u \)
- \( p \rightarrow p_1 \rightarrow \circ \neg r \rightarrow \circ \neg (r \lor s) \)
- \( q \rightarrow \circ \neg (r \lor s) \)
Given $\mathcal{D} = \{T \Rightarrow p; \ p \Rightarrow q; \ q \Rightarrow r\}$ and $\mathcal{F} = \{-r\}$, we can’t derive $p$ and we can’t derive $q$.
Summing up
This Talk

- rule-based approach to defeasible reasoning by cases
- based on the ASPIC framework
- fine-grained control over defeater conditions
- a taste of the many types of problems we encountered
- superior to methods based on manipulating the knowledge base (?)
Thank you