Tackling Defeasible Reasoning in Bochum: the Research Group for Non-Monotonic Logic and Formal Argumentation

Christian Straßer and Dunja Šešelja
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The NMLFA

Reasoning by Cases

Unrestricted Rebut

Comparative Studies

Sequent-based argumentation (with Ofer Arieli, Tel Aviv)

Agent-Based Models
The NMLFA
The Research Group for Non-Monotonic Logic and Formal Argumentation (NMLFA)

- **funding:** 2015–2019 (Alexander von Humboldt-Foundation)
- **aim:** study defeasible reasoning with methods of formal argumentation
- **location:** Institute for Philosophy II, Ruhr-University Bochum
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Members

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- Jesse Heyninck (PhD candidate)
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Some research themes

1. Extending the expressive power of structured argumentation
   1.1 reasoning by cases and hypothetical reasoning
   1.2 expressing doubt – non-greedy argumentative reasoning
   1.3 unrestricted rebut

2. Comparative studies of different nonmonotonic formalisms with special attention to argumentation formalisms (ASPIC, ABA, etc.)

3. Applications of argumentation theory to deontic logic

4. Sequent-based argumentation (with Ofer Arieli, Tel Aviv)

5. Agent-based models based on techniques from abstract argumentation
Reasoning by Cases
Reasoning by Cases, Defeasibly

- strict rules ("→") vs. defeasible rules ("⇒")

Read $A \Rightarrow B$: "$C$ follows defeasibly from $A"$ or "There is a (defeasible) argument for $C$ based on $A"$."


Reasoning by Cases, Defeasibly

- strict rules ("→") vs. defeasible rules ("⇒")
- schematically:

\[
\begin{array}{c}
A \lor B \\
A \Rightarrow C \\
B \Rightarrow C \\
\hline
C
\end{array}
\]
Reasoning by Cases, Defeasibly

- strict rules (“→”) vs. defeasible rules (“⇒”)
- schematically:
  
  \[
  \frac{A \lor B}{C} \quad \frac{A \Rightarrow C}{C} \quad \frac{B \Rightarrow C}{C}
  \]

- or, more generally:
  
  \[
  \frac{A \lor B}{C} \quad \frac{A \Rightarrow \cdots \Rightarrow C}{C} \quad \frac{B \Rightarrow \cdots \Rightarrow C}{C}
  \]
Reasoning by Cases, Defeasibly

• strict rules (“→”) vs. defeasible rules (“⇒”)
• schematically:

\[
\frac{A \lor B \quad A \Rightarrow C \quad B \Rightarrow C}{\therefore C}
\]

• or, more generally:

\[
\frac{A \lor B \quad A \Rightarrow \cdots \Rightarrow C \quad B \Rightarrow \cdots \Rightarrow C}{\therefore C}
\]

• or, more generally:

\[
\frac{A \lor B \quad A \not\Rightarrow C \quad B \not\Rightarrow C}{\therefore C}
\]

Read \( A \not\Rightarrow C \): “\( C \) follows defeasibly from \( A \)” or “There is a (defeasible) argument for \( C \) based on \( A \).”
Are there good formal accounts of defeasible Reasoning by Cases?
• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]
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\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]

• Illustration:
The Meta-Rule Approach: OR

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  \frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad [\text{OR}]
  \]

- Illustration:
  1. \( A \Rightarrow C \)  
     
     PREM
• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad [\text{OR}]
\]

• Illustration:

1. \( A \Rightarrow C \)  \(\text{PREM}\)
2. \( B \Rightarrow C \)  \(\text{PREM}\)
The Meta-Rule Approach: OR

- Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad [\text{OR}]
\]

- Illustration:

1. \( A \Rightarrow C \)  
   PREM
2. \( B \Rightarrow C \)  
   PREM
3. \( A \lor B \)  
   PREM
The Meta-Rule Approach: OR

• Rules for rules:

\[
\frac{A \Rightarrow C \quad B \Rightarrow C}{A \lor B \Rightarrow C} \quad \text{[OR]}
\]

• Illustration:

1. \(A \Rightarrow C\) \hspace{2cm} \text{PREM}
2. \(B \Rightarrow C\) \hspace{2cm} \text{PREM}
3. \(A \lor B\) \hspace{2cm} \text{PREM}
4. \(A \lor B \Rightarrow C\) \hspace{2cm} 1,2; \text{ OR}
The Meta-Rule Approach: OR

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3. \( A \lor B \) \hspace{2cm} \text{PREM}
4. \( A \lor B \Rightarrow C \) \hspace{2cm} 1,2; OR
5. \( C \) \hspace{2cm} 3,4; DefeasibleMP
Suppose we have
\[ \Sigma = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p \} \].
A Problematic Example for OR

\[ p \rightarrow q \lor r \]

\[ q \rightarrow s \rightarrow v \]

\[ r \rightarrow u \rightarrow v \]

\[ v \rightarrow \text{(Right-Weakening)} \] from \( q \rightarrow s \) and \( r \rightarrow u \) by \( (\lor) \)

\[ v \rightarrow \text{(OR)} \] from \( q \lor r \rightarrow v \)
• by \((\text{OR})\): from \(s \Rightarrow v\) and \(u \Rightarrow v\)
A Problematic Example for OR

- by (OR): from $s \Rightarrow v$ and $u \Rightarrow v$
- by (Right-Weakening), from $q \Rightarrow s$ and $r \Rightarrow u$
A Problematic Example for OR

• by (OR): from \( s \Rightarrow v \) and \( u \Rightarrow v \)
• by (Right-Weakening), from \( q \Rightarrow s \) and \( r \Rightarrow u \)
• by (OR): from \( q \Rightarrow s \lor u \) and \( r \Rightarrow s \lor u \)
A Problematic Example for OR

- Suppose now we also have \( t \) and \( t \Rightarrow \neg s \).
- the possible defeater has no effect on the generalized path
A Problematic Example for OR

Suppose now we also have \( t' \) and \( t' \Rightarrow \neg r \).

- the additional possible defeater has no effect on the generalized path.
Extension-based Approaches: Default Logic (Reiter)

- **Input**: set of defaults and a set of formulas ("facts")
- Build *extensions* by applying Modus Ponens to defaults while maintaining consistency
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• Build **extensions** by applying Modus Ponens to defaults while maintaining consistency
• For instance:

![Diagram](image-url)

**Extensions:**
1. \( f \) Nixon; Republican; Quaker; Pacifist
2. \( g \) Nixon; Republican; Quaker; \( \neg \) Pacifist
Extension-based Approaches: Default Logic (Reiter)

- **Input**: set of defaults and a set of formulas (“facts”)
- **Build extensions** by applying Modus Ponens to defaults while maintaining consistency
- For instance:

```plaintext
Extensions:
1. \{\text{Nixon}, \text{Republican}, \text{Quaker}, \neg \text{Pacifist}\}
2. \{\text{Nixon}, \text{Republican}, \text{Quaker}, \text{Pacifist}\}
```
Extension-based Approaches: Default Logic (Reiter)

- no handling of disjunctive facts “out-of-the-box”
- for instance: \( \Sigma = \{ \text{Republican} \lor \text{Democrat}, \text{Republican} \Rightarrow \text{political}, \text{Democrat} \Rightarrow \text{political} \} \).

- since the default is not triggered by the fact, MP cannot be applied
Extension-based Approaches: Default Logic (Reiter)

- idea: split the factual part of the knowledge base (Gelfond, Lifschitz, Przymusinska, 1991)
Extension-based Approaches: Default Logic (Reiter)

- idea: split the factual part of the knowledge base (Gelfond, Lifschitz, Przymusinska, 1991)

- two extensions:
  1. Republican, political
  2. Democrat, political
Consider the following example:

1. Either his left hand or his right hand is broken. \( \text{lhb} \lor \text{rhb} \)
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1. Either his left hand or his right hand is broken. \( \text{lhb} \lor \text{rhb} \)
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Consider the following example:

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3. He writes legibly. \( \text{wl} \)
Consider the following example:

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2. If somebody writes legibly then usually the right hand is not broken. \( \text{wl} \Rightarrow \neg \text{rhb} \)
3. He writes legibly. \( \text{wl} \)

With disjunctive default logic we get two extensions:

1. \( \text{wl}, \neg \text{rhb}, \text{lhb} \)
2. \( \text{wl}, \text{rhb} \)
So far: Manipulate the database!

1. produce new defeasible rules from the given ones
2. produce new factual knowledge bases when confronted with disjunctive information
Enters: the Argumentative Approach

- Instead of manipulating the knowledge base and reasoning on top of the manipulated database,
- we will, in what follows, use a more direct approach to the modeling of Reasoning by Cases in the context of defeasible reasoning, following the inference scheme:

\[
A \lor B \quad A \Rightarrow \cdots \Rightarrow C \quad B \Rightarrow \cdots \Rightarrow C
\]

\[
\frac{\quad C}{\quad C}
\]

or, more generally:

\[
A \lor B \quad A \not\Rightarrow C \quad B \not\Rightarrow C
\]

\[
\frac{\quad C}{\quad C}
\]

- This will allow us to have more control over defeating conditions ...
- ... and to avoid pitfalls as the ones demonstrated above.
Basic idea: Given

- an argument $a_1 \in \text{Arg}(T)$ for which $\text{Conc}(a_1) = A \lor B$,
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- an argument $a_2 \in \text{Arg}(\langle D, K \cup \{A\} \rangle)$ with $\text{Conc}(a_2) = C$, and
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- an argument $a_3 \in \text{Arg}(\langle D, K \cup \{B\}\rangle)$ with $\text{Conc}(a_3) = C$,
Basic idea: Given

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- an argument $a_2 \in \text{Arg}(\langle D, K \cup \{A\} \rangle)$ with $\text{Conc}(a_2) = C$, and
- an argument $a_3 \in \text{Arg}(\langle D, K \cup \{B\} \rangle)$ with $\text{Conc}(a_3) = C$,

we introduce a new RbC-Argument $\langle a_1, [a_2], [a_3] \leadsto C \rangle$. 
More general: RbC-Argument

Definition

Where

- $a_0 \in \text{Arg}(T)$ with $\text{Conc}(a_0) = \bigvee_{i=1}^{n} A_i$ and
- $a_i \in \text{Arg}((\langle D, K \cup \{A_i\} \rangle) \setminus \text{Arg}(T))$ (1 ≤ i ≤ n),

$\langle a_0, [a_1], \ldots, [a_n] \sim \bigvee_{i=1}^{n} \text{Conc}(A_i) \rangle$ is an RbC-argument.
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Where

- \( a_0 \in \text{Arg}(T) \) with \( \text{Conc}(a_0) = \bigvee_{i=1}^{n} A_i \) and
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\( \langle a_0, [a_1], \ldots, [a_n] \rangle \sim \bigvee_{i=1}^{n} \text{Conc}(A_i) \rangle \) is an RbC-argument.

- We say that \( a_1, \ldots, a_n \) are hypothetical sub-arguments of \( a \), in signs: \( a_1, \ldots, a_n \in \text{HSub}(a) \).
More general: RbC-Argument

Definition

Where

- \( a_0 \in \text{Arg}(T) \) with \( \text{Conc}(a_0) = \bigvee_{i=1}^{n} A_i \) and
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\( \langle a_0, [a_1], \ldots, [a_n] \sim \bigvee_{i=1}^{n} \text{Conc}(A_i) \rangle \) is an RbC-argument.

- We say that \( a_1, \ldots, a_n \) are hypothetical sub-arguments of \( a \), in signs: \( a_1, \ldots, a_n \in \text{HSub}(a) \).
- For each \( a_i \), \( \text{Hyp}(a_i) = A_i \).
Let \( T = \langle \mathcal{D}, \mathcal{K} \rangle \) consist of 
\( \mathcal{D} = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \} \) and 
\( \mathcal{K} = \{ p, t \} \).
Let $T = \langle D, K \rangle$ consist of
$D = \{ p \Rightarrow q \lor r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s \}$ and $K = \{ p, t \}$.

We have for instance the arguments:

- $a_1 = \langle \langle p \rangle \Rightarrow q \lor r \rangle \in \text{Arg}(T)$
- $a_2 = \langle \langle q \rangle \Rightarrow s \Rightarrow v \rangle \in \text{Arg}(\langle D, K \cup \{ q \} \rangle)$
- $a_3 = \langle \langle r \rangle \Rightarrow u \Rightarrow v \rangle \in \text{Arg}(\langle D, K \cup \{ r \} \rangle)$
- $a_4 = \langle a_1, [a_2], [a_3] \sim v \rangle \in \text{Arg}(T)$. 
What about attacks?
• $a_1 = \langle \langle p \Rightarrow q \lor r \rangle \rangle \in \text{Arg}(T)$
• $a_2 = \langle \langle q \Rightarrow s \Rightarrow v \rangle \rangle \in \text{Arg}(\langle S, D, K \cup \{q\} \rangle)$
• $a_3 = \langle \langle r \Rightarrow u \Rightarrow v \rangle \rangle \in \text{Arg}(\langle S, D, K \cup \{r\} \rangle)$
• $a_4 = \langle a_1, [a_2], [a_3] \sim v \rangle \in \text{Arg}(T)$
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• $a_3 = \langle \langle r \rangle \Rightarrow u \Rightarrow v \rangle \in \text{Arg}(\langle S, D, K \cup \{r\} \rangle)$
• $a_4 = \langle a_1, [a_2], [a_3] \leadsto v \rangle \in \text{Arg}(T)$
• $a_5 = \langle \langle t \rangle \Rightarrow \neg s \rangle \in \text{Arg}(T)$
\[ a_1 = \langle \langle p \rangle \Rightarrow q \lor r \rangle \in \text{Arg}(T) \]
\[ a_2 = \langle \langle q \rangle \Rightarrow s \Rightarrow v \rangle \in \text{Arg}(\langle D, K \cup \{q\} \rangle) \]
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\[ a_3 = \langle \langle r \Rightarrow u \Rightarrow v \rangle \rangle \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{r\} \rangle) \]
\[ a_4 = \langle a_1, [a_2], [a_3] \Rightarrow v \rangle \in \text{Arg}(T) \]
\[ a_6 = \langle \langle q \Rightarrow \neg s \rangle \rangle \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{q\} \rangle) \]
Attacks again (non-nested case)

Let $\text{HArg}(T)$ be the set of all arguments $a$ for which there is a $b \in \text{Arg}(T)$ for which $a \in \text{HSub}(b)$.
Attacks again (non-nested case)

Let \( \text{HArg}(T) \) be the set of all arguments \( a \) for which there is a \( b \in \text{Arg}(T) \) for which \( a \in \text{HSub}(b) \).

**Argumentation frameworks** are now triples \( \langle \text{Arg}(T), \text{HArg}(T), \text{Attacks}(T) \rangle \).
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Altogether attacks are defined as follows:

\[
\text{Attacks}(T) \subseteq (\text{Arg}(T) \times \text{Arg}(T)) \\
\cup (\text{Arg}(T) \times \text{HArg}(T)) \\
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\]

where \( a \) rebuts \( b = \langle b_1, \ldots, b_n \Rightarrow B \rangle \) iff \( \text{Conc}(a) = \neg B \) or \( B = \neg \text{Conc}(a) \) and

1. \( a \in \text{Arg}(T) \) and \( b \in \text{Arg}(T) \cup \text{HArg}(T) \) or
Attacks again (non-nested case)

Let $HArg(T)$ be the set of all arguments $a$ for which there is a $b \in \text{Arg}(T)$ for which $a \in HSub(b)$.

**Argumentation frameworks** are now triples $\langle \text{Arg}(T), HArg(T), \text{Attacks}(T) \rangle$.

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where $a$ rebuts $b = \langle b_1, \ldots, b_n \Rightarrow B \rangle$ iff $\text{Conc}(a) = \neg B$ or $B = \neg \text{Conc}(a)$ and

1. $a \in \text{Arg}(T)$ and $b \in \text{Arg}(T) \cup HArg(T)$ or
2. $a \in HArg(T)$ and $b \in HArg(T)$ and $\text{Hyp}(a) = \text{Hyp}(b)$. 
Unrestricted Rebut
joint work: Jesse Heyninck and Christian Straßer
• in ASPIC\(^+\) only \textit{restricted} rebut: \(a\) rebuts \(b\) iff
  1. the conclusion of \(a\) is contrary to the conclusion of \(b\)
  2. and \(b\) has a defeasible top rule
Status quo

• in ASPIC$^+$ only **restricted** rebut: $a$ rebuts $b$ iff
  1. the conclusion of $a$ is contrary to the conclusion of $b$
  2. and $b$ has a defeasible top rule

• unrestricted rebut: only the first requirement
  • **pro**: natural (Caminada)
  • **contra**: leads to trouble for many semantics such as preferred, stable, etc
Enters: Caminada et al. (COMMA 2014)

- for grounded semantics unrestricted rebut works just fine (really?)
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• for grounded semantics unrestricted rebut works just fine (really?)
• Rationality postulates:
  • Sub-argument closure: where $a \in \mathcal{E}$ and $b \in \text{Sub}(a)$, $b \in \mathcal{E}$. 
• for grounded semantics unrestricted rebut works just fine (really?)

• Rationality postulates:
  • **Sub-argument closure**: where \( a \in \mathcal{E} \) and \( b \in \mathrm{Sub}(a), b \in \mathcal{E} \).
  • **Closure under strict rules**: where \( a_1, \ldots, a_n \in \mathcal{E} \cap \mathrm{Arg}(T) \) and \( \mathrm{Conc}(a_1), \ldots, \mathrm{Conc}(a_n) \vdash B \), also \( \langle a_1, \ldots, a_n \rightarrow B \rangle \in \mathcal{E} \cap \mathrm{Arg}(T) \)
• for grounded semantics unrestricted rebut works just fine (really?)
• Rationality postulates:
  • Sub-argument closure: where $a \in \mathcal{E}$ and $b \in \text{Sub}(a)$, $b \in \mathcal{E}$.
  • Closure under strict rules: where $a_1, \ldots, a_n \in \mathcal{E} \cap \text{Arg}(T)$ and $\text{Conc}(a_1), \ldots, \text{Conc}(a_n) \vdash B$, also $\langle a_1, \ldots, a_n \rightarrow B \rangle \in \mathcal{E} \cap \text{Arg}(T)$
  • Consistency: $\{\text{Conc}(a) \mid a \in \mathcal{E}\}$ is consistent.
Grounded Semantics

- first select unattacked arguments
- remove the arguments attacked by the selected arguments
- select unattacked arguments
- remove the arguments attacked by the selected arguments
- and so on… until fixed point is reached
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Grounded Semantics

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For a set of formulas $\mathcal{F}$ let $\text{Atoms}(\mathcal{F})$ be the set of all propositional atoms in $\mathcal{F}$.

\(^1\text{Caminada, Carnielli, Dunne (JLC, 2012). Avron (2016) calls this the basic relevance criterion.}\)
For a set of formulas $\mathcal{F}$ let $\text{Atoms}(\mathcal{F})$ be the set of all propositional atoms in $\mathcal{F}$.

- Non-interference\(^1\) Where $T = \langle D, K \rangle$ and $T' = \langle D', K' \rangle$ are argumentation theories and $A$ is a formula such that $\text{Atoms}(D \cup K \cup \{A\}) \cap \text{Atoms}(D' \cup K') = \emptyset$ then:

$$T \vdash A \iff \langle D \cup D', K \cup K' \rangle \vdash A.$$  

\(^1\)Caminada, Carnielli, Dunne (JLC, 2012). Avron (2016) calls this the basic relevance criterion.
The problem with unrestricted rebut in ASPIC⁻

- Take the knowledge base: \( \{ \top \Rightarrow p \} \).

- Clearly: \( a = \langle \top \Rightarrow p \rangle \) is in the grounded extension.

- Now, take the knowledge base: \( \{ \top \Rightarrow p; \top \Rightarrow s; \top \Rightarrow s \} \).

- (Let the strict rules be closed under classical logic.)

- Now, \( a \) is attacked by \( \langle \langle \top \Rightarrow s \rangle; \langle \top \Rightarrow s \rangle \rangle !: p \rangle \).

- As a consequence, \( a \) is not in the grounded extension.

- Thus, Non-Interference doesn't hold for unrestricted rebut.
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The problem with unrestricted rebut in ASPIC⁻

- Take the knowledge base: \( \{ T \Rightarrow p \} \).
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The problem with unrestricted rebut in ASPIC

- Take the knowledge base: \{\top \Rightarrow p\}.
- Clearly: \(a = \langle \top \Rightarrow p \rangle\) is in the grounded extension.
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The problem with unrestricted rebut in ASPIC−

• Take the knowledge base: \( \{ T \Rightarrow p \} \).
• Clearly: \( a = \langle T \Rightarrow p \rangle \) is in the grounded extension.
• Now, take the knowledge base: \( \{ T \Rightarrow p, T \Rightarrow s, T \Rightarrow \neg s \} \).
• (Let the strict rules be closed under classical logic.)
• Now, \( a \) is attacked by \( \langle \langle T \Rightarrow s \rangle, \langle T \Rightarrow \neg s \rangle \Rightarrow \neg p \rangle \).
The problem with unrestricted rebut in ASPIC

• Take the knowledge base: \{T ⇒ p\}.
• Clearly: \(a = \langle T ⇒ p \rangle\) is in the grounded extension.
• Now, take the knowledge base: \{T ⇒ p, T ⇒ s, T ⇒ ¬s\}.
• (Let the strict rules be closed under classical logic.)
• Now, \(a\) is attacked by \(\langle \langle T ⇒ s \rangle, \langle T ⇒ ¬s \rangle ⇒ ¬p \rangle\).
• As a consequence, \(a\) is not in the grounded extension.
The problem with unrestricted rebut in ASPIC⁻

- Take the knowledge base: \( \{T \Rightarrow p\} \).
- Clearly: \( a = \langle T \Rightarrow p \rangle \) is in the grounded extension.
- Now, take the knowledge base: \( \{T \Rightarrow p, T \Rightarrow s, T \Rightarrow \neg s\} \).
- (Let the strict rules be closed under classical logic.)
- Now, \( a \) is attacked by \( \langle \langle T \Rightarrow s \rangle, \langle T \Rightarrow \neg s \rangle \Rightarrow \neg p \rangle \).
- As a consequence, \( a \) is not in the grounded extension.
- Thus, Non-Interference doesn’t hold for unrestricted rebut.
Prima Facie solution

- sort out inconsistent arguments (Wu, 2012: this works in ASPIC+)
- however, this doesn’t work with unrestricted rebut
Prima Facie solution ii

Let \( \{ T \Rightarrow_1 p, p \Rightarrow_1 q, T \Rightarrow_2 \neg(p \land q) \} \) be our knowledge base.

We have, e.g., the following arguments:

- \( a = \langle T \Rightarrow_1 p \rangle \)
- \( b = \langle a \Rightarrow_1 q \rangle \)
- \( a \oplus b = \langle a, b \rightarrow p \land q \rangle \)
- \( c = \langle T \Rightarrow_2 \neg(p \land q) \rangle \)
- \( a \oplus c = \langle a, c \rightarrow \neg q \rangle \)
- \( b \oplus c = \langle b, c \rightarrow \neg p \rangle \)
Let \( \{ \top \Rightarrow_1 p, p \Rightarrow_1 q, \top \Rightarrow_2 \neg (p \land q) \} \) be our knowledge base.

We have, e.g., the following arguments:

- \( a = \langle \top \Rightarrow_1 p \rangle \)
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- \( a \oplus c = \langle a, c \rightarrow \neg q \rangle \)
- \( b \oplus c = \langle b, c \rightarrow \neg p \rangle \)

Problem:

- \( b \oplus c \) is inconsistent and thus filtered out
- this leaves \( a \) and \( c \) in but \( a \oplus c \) out of the grounded extension.
- Failure of closure!
Enters: ASPIC\textsuperscript{\textcopyright}: generalized unrestricted rebut

- lifting of the contrariness operator to (finite) sets of formulas, e.g.,
Enters: ASPI^⊕: generalized unrestricted rebut

- lifting of the contrariness operator to (finite) sets of formulas, e.g.,
  - \( \{A_1, \ldots, A_n\} \equiv \bigwedge_{i=1}^{n} A_i \), or

\[ \text{Concs}(a) = \{ \text{Conc}(b) \mid b \in \text{Sub}(a) \} \]
Enters: $\text{ASPIC}^{\ominus}$: generalized unrestricted rebut

- lifting of the contrariness operator to (finite) sets of formulas, e.g.,
  - $\{A_1, \ldots, A_n\} = \text{df } \bigwedge_{i=1}^{n} A_i$, or
  - $\{A_1, \ldots, A_n\} = \text{df } \bigvee_{i=1}^{n} A_i$. 

\[ \text{Conc}(a) = \text{df } \bigcup \text{Sub}(a) \]
• lifting of the contrariness operator to (finite) sets of formulas, e.g.,
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  \[ \{A_1, \ldots, A_n\} = \text{df} \bigvee_{i=1}^{n} A_i. \]
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Enters: ASPIC\(\oplus\): generalized unrestricted rebut

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- Concs\((a) = \text{df} \{\text{Conc}(b) \mid b \in \text{Sub}(a)\}\)

**Definition**

\(a\ \text{gen-rebuts}\ b\ \text{iff}\ \text{b is defeasible and Conc}(a) = \Delta\ \text{for some} \ \Delta \subseteq \text{Concs}(b).\)

**Definition**

\(a\ \text{gen-defeats}\ b\ \text{iff}\ a\ \text{gen-rebuts}\ c\ \text{for some} \ c \in \text{Sub}(b)\ \text{and} \ c \preceq a.\)
Back to the example
Where the strict rules are obtained from classical logic, for weakest link we get

- sub-argument closure
- closure under strict rules
- consistency
- non-interference
Comparative Studies
Jesse Heyninck, Christian Straßer: *Relations between assumption-based approaches in nonmonotonic logic and formal argumentation* (NMR 2016, Cape Town, also available on Arxiv)
The landscape

- ABA: assumption-based argumentation (Dung, Kowalski, Toni)
- ALs: adaptive logics (Batens)
- DACR: default assumptions (Makinson)
- KLM: preferential semantics (Shoham, Kraus/Lehman/Magidor)
On the argumentative side

- **ASPIC⁺**: 
  - defeasible and strict rules
  - various attack types: rebuts, undercuts, undermine
  - arguments as proof trees

- **ABA (Assumption-based argumentation)**: 
  - only strict rules
  - higher level of abstraction: arguments as sets of defeasible assumptions
  - only "assumption-attacks" (undermine)
  - our translation: without priorities
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- our translation: without priorities
Currently

- extending (Heyninck, Straßer, NMR 2016) with priorities
- relations between adaptive logics and parametrized logic programming (Jesse Heyninck, Pere Pardo, Christian Straßer)
Sequent-based argumentation (with Ofer Arieli, Tel Aviv)
Sequent-based Argumentation

- Arguments are $\Gamma_1 \Rightarrow \varphi$,
- $\Gamma_2 \Rightarrow \neg \varphi$,
- $\Gamma_3, \neg \varphi \Rightarrow \psi$,
- $\Gamma_4, \neg \psi \Rightarrow \psi'$.
• arguments are $\mathcal{C}$-provable sequents, where
• arguments are $\mathcal{C}$-provable sequents, where
• $\mathcal{C}$ is a sound and complete sequent-calculus
Sequent-based Argumentation

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- $\mathcal{C}$ is a sound and complete sequent-calculus
- of a (Tarskian) core logic $\mathbb{L}$
Sequent-based attacks: elimination rules

<table>
<thead>
<tr>
<th>Attacker Sequent</th>
<th>Conditions</th>
<th>Attacked Sequent</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Eliminated Sequent</td>
</tr>
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Examples

• **Undercut:**

  \[
  1 \\
  1 ; 2 ; 2 ; 2 \quad \land \quad 2 ; 2 ; 2 \quad \not= \\
  \]

• **Compact Undercut:**

  \[
  1 \\
  1 ) : \land \quad 2 ; 2 ; 2 \quad \not= \\
  \]

• **Rebuttal:**

  \[
  1 \\
  1 ) \\
  1 ) : 2 \\
  2 ; 2 ; 2 \quad \not= \\
  \]

• **Specificity, etc.**
Sequent-based attacks: elimination rules

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<td>( \Gamma_2, \Gamma_2' \nRightarrow \psi_2 )</td>
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Examples

- Undercut:  
  \[ \Gamma_1 \Rightarrow \psi_1 \Rightarrow \psi_1 \leftarrow -\Gamma_2' \rightleftharpoons \Gamma_2, \Gamma_2' \Rightarrow \psi_2 \]
### Sequent-based attacks: elimination rules

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<td>• Compact Undercut:</td>
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### Examples

- **Undercut:**
  \[
  \Gamma_1 \Rightarrow \psi_1 \quad \Rightarrow \psi_1 \leftrightarrow -\bigwedge \Gamma'_2 \quad \Gamma_2, \Gamma'_2 \Rightarrow \psi_2 \\
  \Gamma_2, \Gamma'_2 \not\Rightarrow \psi_2
  \]

- **Compact Undercut:**
  \[
  \Gamma_1 \Rightarrow -\bigwedge \Gamma'_2 \quad \Gamma_2, \Gamma'_2 \Rightarrow \psi \\
  \Gamma_2, \Gamma'_2 \not\Rightarrow \psi
  \]

- **Rebuttal:**
  \[
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Examples

• Undercut:

• Compact Undercut:

• Rebuttal:

• Specificity, etc.
Dynamic proof theories

1. \( p \Rightarrow p \) \quad \text{Axiom}
2. \( \Rightarrow p, \neg p \) \quad [\Rightarrow \neg], 1
3. \( \Rightarrow p \lor \neg p \) \quad [\Rightarrow \lor], 2
4. \( p \lor \neg p \Rightarrow \neg (p \land \neg p) \) \quad \ldots
5. \( \neg (p \land \neg p) \Rightarrow p \lor \neg p \) \quad \ldots
6. \( q \Rightarrow q \) \quad \text{Axiom}
7. \( \neg p \Rightarrow \neg p \) \quad \text{Axiom}
8. \( p \not\Rightarrow p \) \quad \text{Ucut, 7, 7, 7, 1} \quad \neg p \Rightarrow \neg p
9. \( p \Rightarrow \neg \neg p \) \quad \ldots
10. \( \neg \neg p \Rightarrow p \) \quad \ldots
11. \( \neg p \not\Rightarrow \neg p \) \quad \text{Ucut, 1, 9, 10, 7} \quad p \Rightarrow p
function Evaluate($D$) /* $D$
Attack := ∅; Elim := ∅; Derived := ∅;
while ($D$ is not empty) do {
    if (Top($D$) = $\langle i, s, J, \emptyset \rangle$) then
        Derived := Derived $\cup \{s\}$;
    if (Top($D$) = $\langle i, \bar{s}, J, r \rangle$) then
        if ($r \notin$ Elim) then
            Elim := Elim $\cup \{s\}$ and
            Attack := Attack $\cup \{r\}$;
    $D$ := Tail($D$); }
Accept := Derived $-$ Elim;
return (Attack, Elim, Accept)

• A derivation must be coherent: Attack($D$) $\cap$ Elim($D$) = ∅
function Evaluate($\mathcal{D}$) /* $\mathcal{D}$
Attack := $\emptyset$; Elim := $\emptyset$; Derived := $\emptyset$;
while ($\mathcal{D}$ is not empty) do { 
  if (Top($\mathcal{D}$) = $\langle i, s, J, \emptyset \rangle$) then 
    Derived := Derived $\cup$ \{s\};
  if (Top($\mathcal{D}$) = $\langle i, \bar{s}, J, r \rangle$) then 
    if ($r \not\in$ Elim) then 
      Elim := Elim $\cup$ \{s\} and 
      Attack := Attack $\cup$ \{r\};
  $\mathcal{D}$ := Tail($\mathcal{D}$); }
Accept := Derived $\setminus$ Elim;
return (Attack, Elim, Accept)

• A derivation must be coherent: $\text{Attack}(D) \cap \text{Elim}(D) = \emptyset$
• A sequent $A$ is finally derived in a dynamic derivation $D$ if $A \in \text{Accept}(D)$ and $D$ cannot be extended to a dynamic derivation $D'$ such that $A \in \text{Elim}(D')$. 
Sequent-based Argumentation: some publications

Agent-Based Models
• joint work: AnneMarie Borg, Daniel Frey, Dunja Šešelja and Christian Straßer


Explanatory Argumentation Frameworks

Šešelja and Straßer, Synthese, 2013, 190:2195–2217
Abstract argumentation in our ABM

We represent in an abstract way:

- arguments
- discovery relation
- attack relation