

Tackling Defeasible Reasoning in Bochum:

the Research Group for Non-Monotonic Logic and Formal
Argumentation

Christian Straßer and Dunja Šešelja

April 10, 2017

The NMLFA

Reasoning by Cases

Unrestricted Rebut

Comparative Studies

Sequent-based argumentation (with Ofer Arieli, Tel Aviv)

Agent-Based Models

The NMLFA

The Research Group for Non-Monotonic Logic and Formal Argumentation (NMLFA)

Research Group for
Non-Monotonic Logic
and **Formal Argumentation**



- **funding:** 2015–2019 (Alexander von Humboldt-Foundation)
- **aim:** study defeasible reasoning with methods of formal argumentation
- **location:** Institute for Philosophy II, Ruhr-University Bochum
- **online:**
 - <http://homepage.ruhr-uni-bochum.de/defeasible-reasoning/index.html>
 - <mailto:defeasible-reasoning@rub.de>

Members

- AnneMarie Borg (PhD candidate)



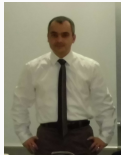
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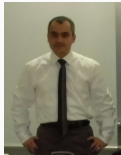
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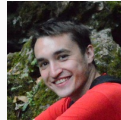
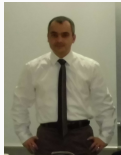
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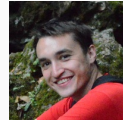
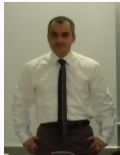
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Some research themes

1. Extending the expressive power of structured argumentation
 - 1.1 reasoning by cases and hypothetical reasoning
 - 1.2 expressing doubt – non-greedy argumentative reasoning
 - 1.3 unrestricted rebut
2. Comparative studies of different nonmonotonic formalisms with special attention to argumentation formalisms (ASPIC, ABA, etc.)
3. Applications of argumentation theory to deontic logic
4. Sequent-based argumentation (with Ofer Arieli, Tel Aviv)
5. Agent-based models based on techniques from abstract argumentation

Reasoning by Cases

Mathieu Beirlaen, Jesse Heyninck, and Christian Straßer,
Reasoning by Cases in Structured Argumentation forthcoming
in Proceedings KRR/SAC 2017, ACM Digital Library (2017)

Reasoning by Cases, Defeasibly

- strict rules (“ \rightarrow ”) vs. defeasible rules (“ \Rightarrow ”)

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- or, more generally:

$$\frac{A \vee B \quad A \vdash\sim C \quad B \vdash\sim C}{C}$$

Read $A \vdash\sim C$: “C follows defeasibly from A” or “There is a (defeasible) argument for C based on A.”

Are there good formal accounts of **defeasible**
Reasoning by Cases?

The Meta-Rule Approach: OR

- Rules for rules:

$$\frac{A \Rightarrow C \quad B \Rightarrow C}{A \vee B \Rightarrow C} \text{ [OR]}$$

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2. $B \Rightarrow C$

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4. $A \vee B \Rightarrow C$	1,2; OR

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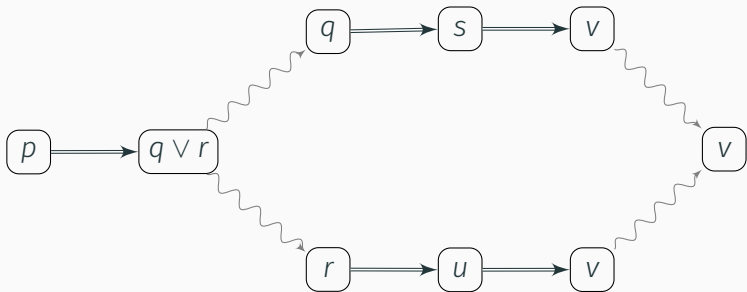
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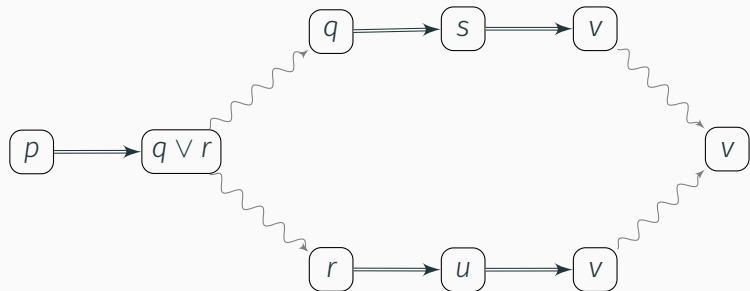
A Problematic Example for OR

Suppose we have

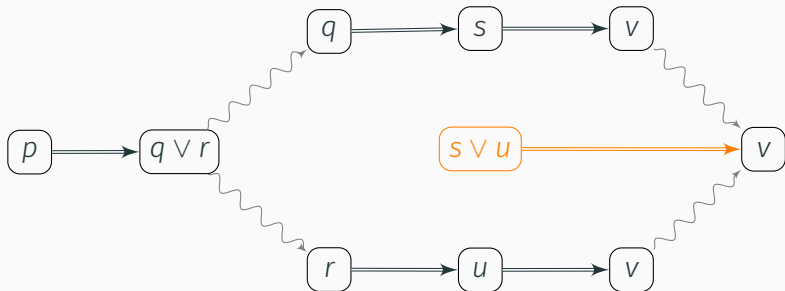
$$\Sigma = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p\}.$$



A Problematic Example for OR

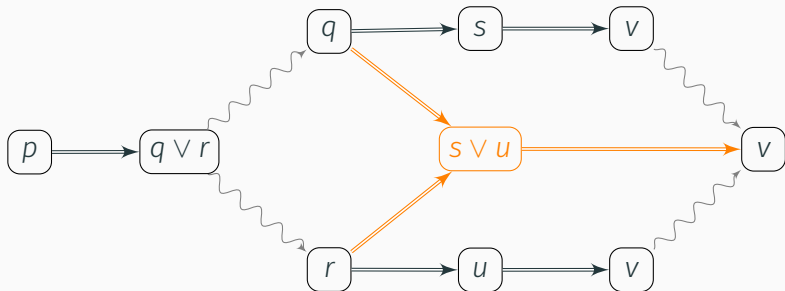


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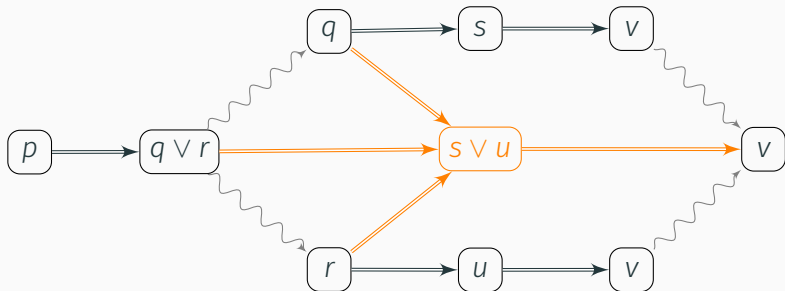
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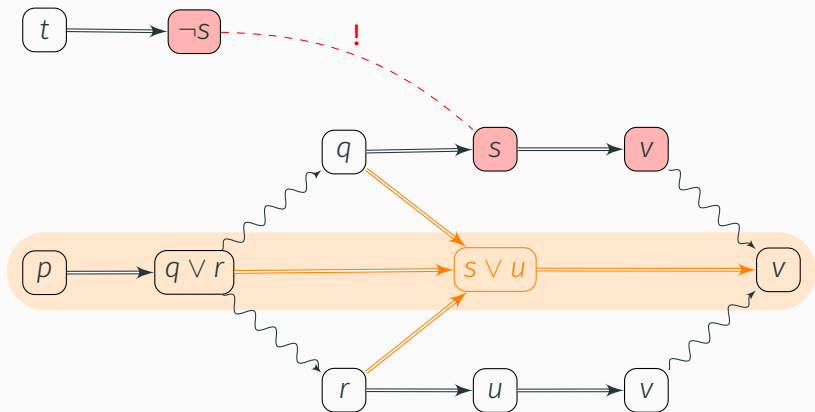
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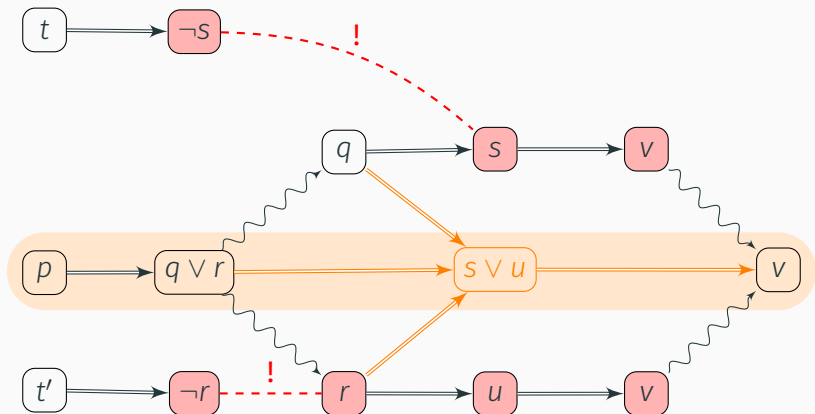
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A Problematic Example for OR



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- the possible defeater has no effect on the generalized path

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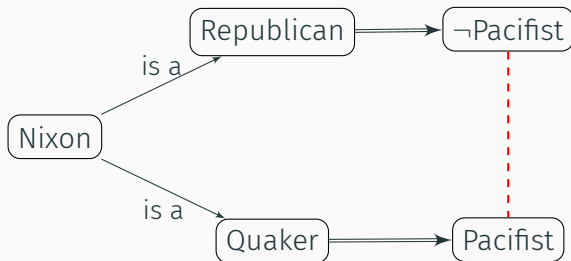
- Suppose now we also have t' and $t' \Rightarrow \neg r$.
- the additional possible defeater has no effect on the generalized path

Extension-based Approaches: Default Logic (Reiter)

- **Input**: set of defaults and a set of formulas (“facts”)
- Build **extensions** by applying Modus Ponens to defaults while maintaining consistency

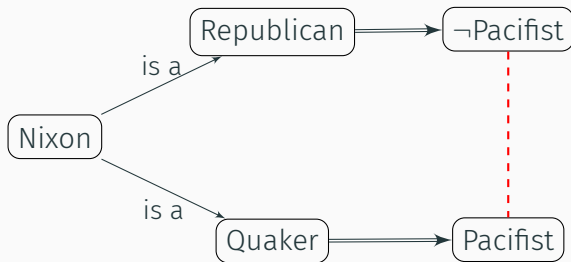
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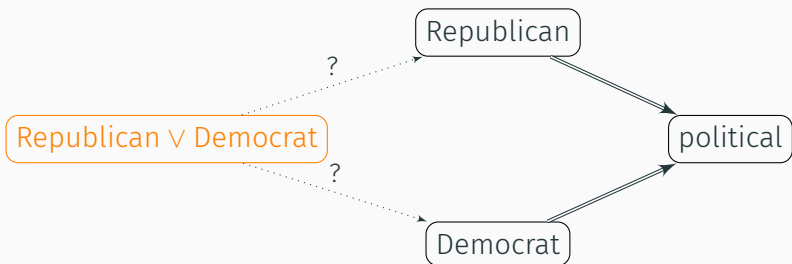
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- Extensions:
 1. {Nixon, Republican, Quaker, ¬Pacifist}
 2. {Nixon, Republican, Quaker, Pacifist}

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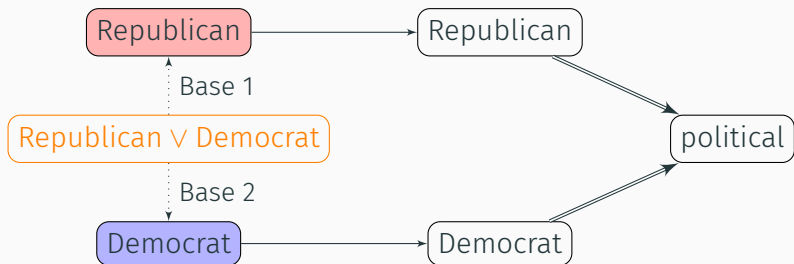
- no handling of disjunctive facts “out-of-the-box”
- for instance: $\Sigma = \{\text{Republican} \vee \text{Democrat}, \text{Republican} \Rightarrow \text{political}, \text{Democrat} \Rightarrow \text{political}\}$.



- since the default is not triggered by the fact, MP cannot be applied

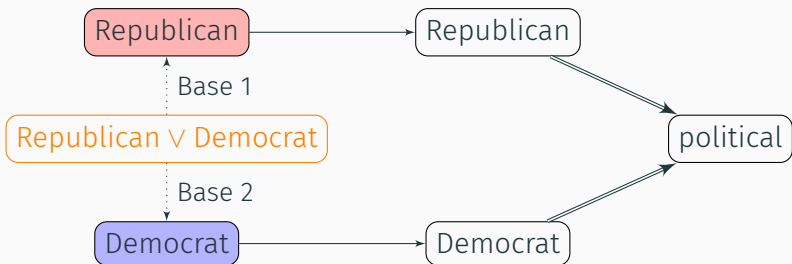
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- two extensions:
 1. Republican, political
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Consider the following example:

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2. If somebody writes legibly then usually the right hand is not broken. $\boxed{wl \Rightarrow \neg rhb}$
3. He writes legibly. \boxed{wl}

With disjunctive default logic we get two extensions:

1. $wl, \neg rhb, lhb$
2. wl, rhb

So far: **Manipulate the database!**

1. produce new defeasible rules from the given ones
2. produce new factual knowledge bases when confronted with disjunctive information

Enters: the Argumentative Approach

- Instead of manipulating the knowledge base and reasoning on top of the manipulated database,
- we will, in what follows, use a **more direct approach** to the modeling of Reasoning by Cases in the context of defeasible reasoning, following the inference scheme:

$$\frac{A \vee B \quad A \Rightarrow \dots \Rightarrow C \quad B \Rightarrow \dots \Rightarrow C}{C}$$

or, more generally:

$$\frac{A \vee B \quad A \sim C \quad B \sim C}{C}$$

- This will allow us to have **more control** over defeating conditions ...
- ... and to avoid pitfalls as the ones demonstrated above.

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we introduce a new RbC-Argument $\langle a_1, [a_2], [a_3] \rightsquigarrow C \rangle$.

More general: RbC-Argument

Definition

Where

- $a_0 \in \text{Arg}(T)$ with $\text{Conc}(a_0) = \bigvee_{i=1}^n A_i$ and
- $a_i \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{A_i\} \rangle) \setminus \text{Arg}(T)$ ($1 \leq i \leq n$),

$\langle a_0, [a_1], \dots, [a_n] \rangle \rightsquigarrow \bigvee_{i=1}^n \text{Conc}(A_i)$ is an RbC-argument.

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- We say that a_1, \dots, a_n are **hypothetical sub-arguments** of a , in signs: $a_1, \dots, a_n \in \text{HSub}(a)$.
- For each a_i , $\text{Hyp}(a_i) = A_i$.

Let $T = \langle \mathcal{D}, \mathcal{K} \rangle$ consist of

$\mathcal{D} = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s\}$ and

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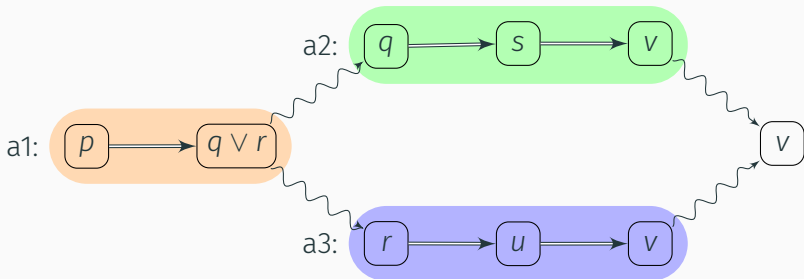
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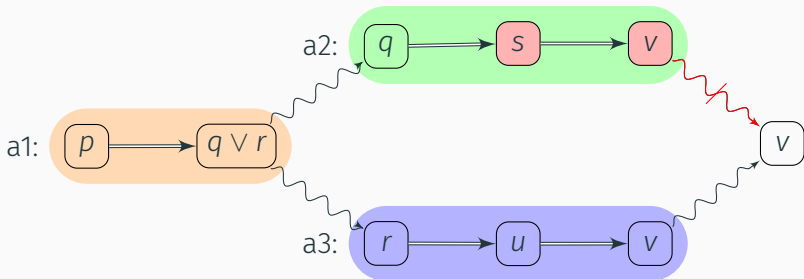
We have for instance the arguments:

- $a_1 = \langle \langle p \rangle \Rightarrow q \vee r \rangle \in \text{Arg}(T)$
- $a_2 = \langle \langle q \rangle \Rightarrow s \Rightarrow v \rangle \in \text{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{q\} \rangle)$
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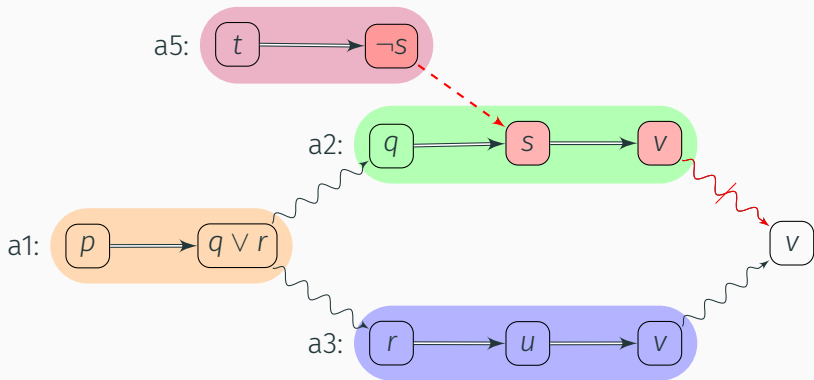


What about attacks?

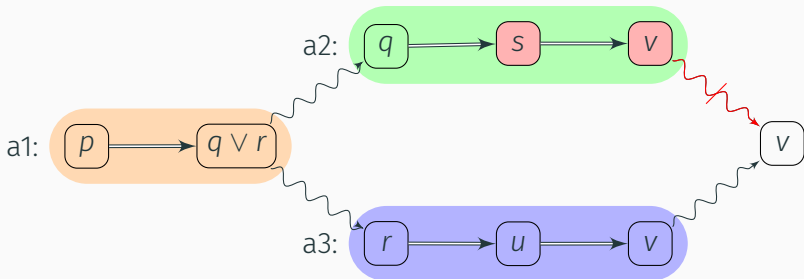
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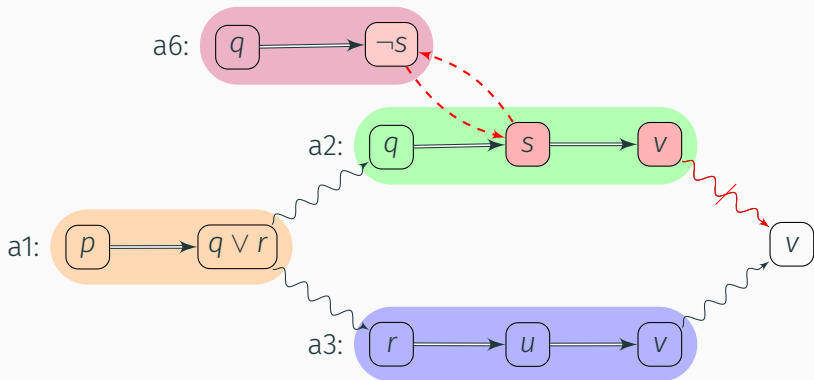
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Attacks again (non-nested case)

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2. $a \in \text{HArg}(T)$ and $b \in \text{HArg}(T)$ and $\text{Hyp}(a) = \text{Hyp}(b)$.

Unrestricted Rebut

joint work: Jesse Heyninck and Christian Straßer

- in ASPIC⁺ only **restricted** rebut: a rebuts b iff
 1. the conclusion of a is contrary to the conclusion of b
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 1. the conclusion of a is contrary to the conclusion of b
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- unrestricted rebut: only the first requirement
 - **pro**: natural (Caminada)
 - **contra**: leads to trouble for many semantics such as preferred, stable, etc

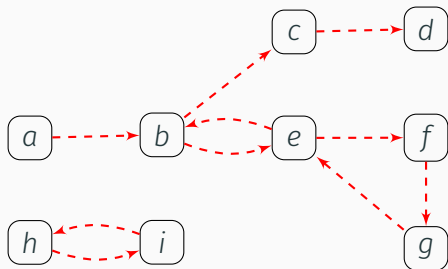
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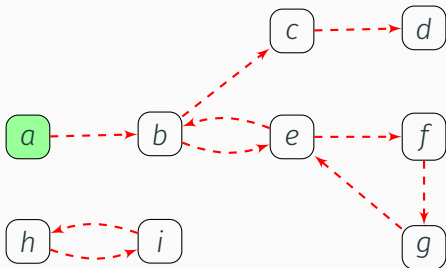
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 - **Consistency**: $\{\text{Conc}(a) \mid a \in \mathcal{E}\}$ is consistent.

Grounded Semantics

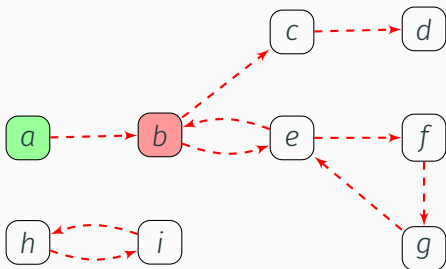


Grounded Semantics



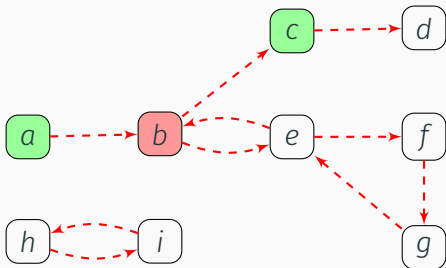
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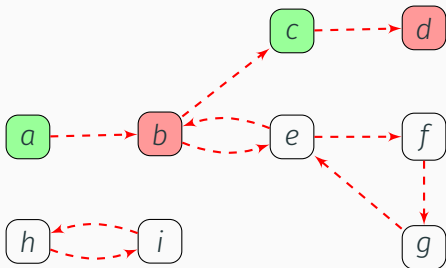
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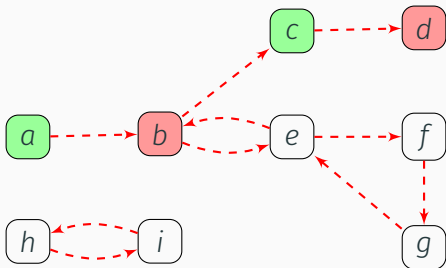
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- and so on ... until fixed point is reached

For a set of formulas \mathcal{F} let $\text{Atoms}(\mathcal{F})$ be the set of all propositional atoms in \mathcal{F} .

¹Caminada, Carnielli, Dunne (JLC, 2012). Avron (2016) calls this the **basic relevance criterion**.

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- **Non-interference**¹ Where $T = \langle \mathcal{D}, \mathcal{K} \rangle$ and $T' = \langle \mathcal{D}', \mathcal{K}' \rangle$ are argumentation theories and A is a formula such that $\text{Atoms}(\mathcal{D} \cup \mathcal{K} \cup \{A\}) \cap \text{Atoms}(\mathcal{D}' \cup \mathcal{K}') = \emptyset$ then:

$$T \vdash A \text{ iff } \langle \mathcal{D} \cup \mathcal{D}', \mathcal{K} \cup \mathcal{K}' \rangle \vdash A.$$

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- Thus, Non-Interference doesn't hold for unrestricted rebut.

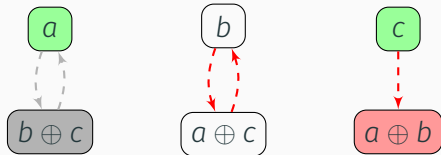
- sort out inconsistent arguments (Wu, 2012: this works in ASPIC⁺)
- however, this doesn't work with unrestricted rebut

Prima Facie solution ii

Let $\{T \Rightarrow_1 p, p \Rightarrow_1 q, T \Rightarrow_2 \neg(p \wedge q)\}$ be our knowledge base.

We have, e.g., the following arguments:

- $a = \langle T \Rightarrow_1 p \rangle$
- $b = \langle a \Rightarrow_1 q \rangle$
- $a \oplus b = \langle a, b \rightarrow p \wedge q \rangle$
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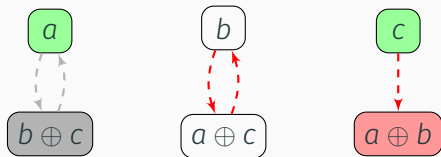


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Problem:

- $b \oplus c$ is inconsistent and thus filtered out
- this leaves a and c in but $a \oplus c$ out of the grounded extension.
- Failure of closure!

Enters: $ASPIC^\ominus$: generalized unrestricted rebut

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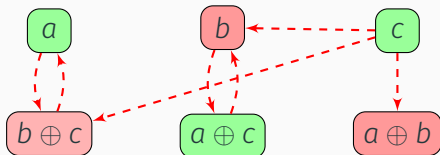
Definition

a **gen-rebuts** b iff b is defeasible and $\text{Conc}(a) = \overline{\Delta}$ for some $\Delta \subseteq \text{Concs}(b)$.

Definition

a **gen-defeats** b iff a gen-rebuts c for some $c \in \text{Sub}(b)$ and $c \preceq a$.

Back to the example



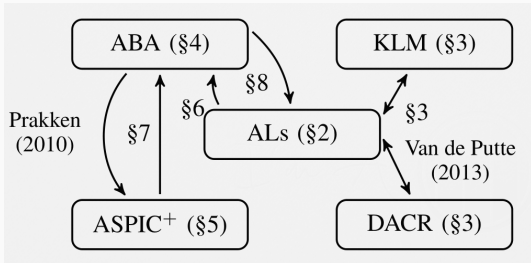
Where the strict rules are obtained from classical logic, for weakest link we get

- sub-argument closure
- closure under strict rules
- consistency
- non-interference

Comparative Studies

Jesse Heyninck, Christian Straßer: *Relations between assumption-based approaches in nonmonotonic logic and formal argumentation* (NMR 2016, Cape Town, also available on Arxiv)

The landscape



- ABA: assumption-based argumentation (Dung, Kowalski, Toni)
- ALs: adaptive logics (Batens)
- DACR: default assumptions (Makinson)
- KLM: preferential semantics (Shoham, Kraus/Lehman/Magidor)

- ASPIC⁺:
 - defeasible and strict rules
 - various attack types: rebuts, undercuts, undermine
 - arguments as proof trees

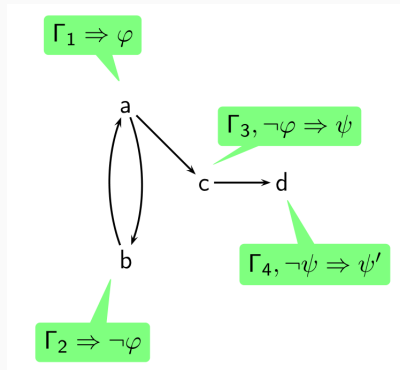
On the argumentative side

- ASPIC⁺:
 - defeasible and strict rules
 - various attack types: rebuts, undercuts, undermine
 - arguments as proof trees
- ABA (Assumption-based argumentation)
 - only strict rules
 - higher level of abstraction: arguments as sets of defeasible assumptions
 - only “assumption-attacks” (\approx undermine)
- our translation: without priorities

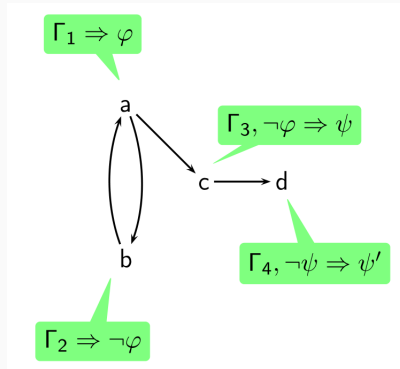
- extending (Heyninck, Straßer, NMR 2016) with priorities
- relations between adaptive logics and parametrized logic programming (Jesse Heyninck, Pere Pardo, Christian Straßer)

Sequent-based argumentation (with Ofer Arieli, Tel Aviv)

Sequent-based Argumentation

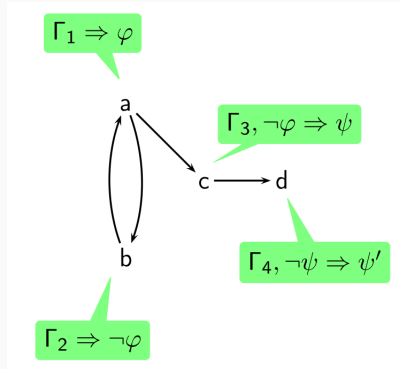


Sequent-based Argumentation



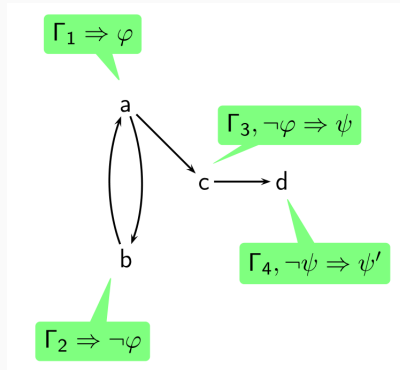
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Sequent-based attacks: elimination rules

Attacker Sequent	Conditions	Attacked Sequent
<hr/>		
Eliminated Sequent		

Sequent-based attacks: elimination rules

Attacker Sequent Conditions Attacked Sequent
Eliminated Sequent

Examples

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$$\frac{\Gamma_1 \Rightarrow \psi_1 \quad \Rightarrow \psi_1 \leftrightarrow \neg \wedge \Gamma'_2 \quad \Gamma_2, \Gamma'_2 \Rightarrow \psi_2}{\Gamma_2, \Gamma'_2 \not\Rightarrow \psi_2}$$

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- Specificity, etc.

Dynamic proof theories

1.	$p \Rightarrow p$	Axiom	
2.	$\Rightarrow p, \neg p$	$[\Rightarrow \neg], 1$	
3.	$\Rightarrow p \vee \neg p$	$[\Rightarrow \vee], 2$	
4.	$p \vee \neg p \Rightarrow \neg(p \wedge \neg p)$...	
5.	$\neg(p \wedge \neg p) \Rightarrow p \vee \neg p$...	
6.	$q \Rightarrow q$	Axiom	
7.	$\neg p \Rightarrow \neg p$	Axiom	
8.	$p \not\Rightarrow p$	Ucut, 7, 7, 7, 1	$\neg p \Rightarrow \neg p$
9.	$p \Rightarrow \neg\neg p$...	
10.	$\neg\neg p \Rightarrow p$...	
11.	$\neg p \not\Rightarrow \neg p$	Ucut, 1, 9, 10, 7	$p \Rightarrow p$

Dynamic proof theories (Retraction, Basic Idea)

```
function Evaluate( $\mathcal{D}$ ) /*  $\mathcal{D}$ 
Attack :=  $\emptyset$ ; Elim :=  $\emptyset$ ; Derived :=  $\emptyset$ ;
while ( $\mathcal{D}$  is not empty) do {
    if (Top( $\mathcal{D}$ ) =  $\langle i, s, J, \emptyset \rangle$ ) then
        Derived := Derived  $\cup$  { $s$ };
    if (Top( $\mathcal{D}$ ) =  $\langle i, \bar{s}, J, r \rangle$ ) then
        if ( $r \notin$  Elim) then
            Elim := Elim  $\cup$  { $s$ } and
            Attack := Attack  $\cup$  { $r$ };
     $\mathcal{D} :=$  Tail( $\mathcal{D}$ ); }
Accept := Derived – Elim;
return (Attack, Elim, Accept)
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- A derivation must be *coherent*: $\text{Attack}(D) \cap \text{Elim}(D) = \emptyset$
- A sequent A is **finally derived** in a dynamic derivation D if $A \in \text{Accept}(D)$ and D cannot be extended to a dynamic derivation D' such that $A \in \text{Elim}(D')$.

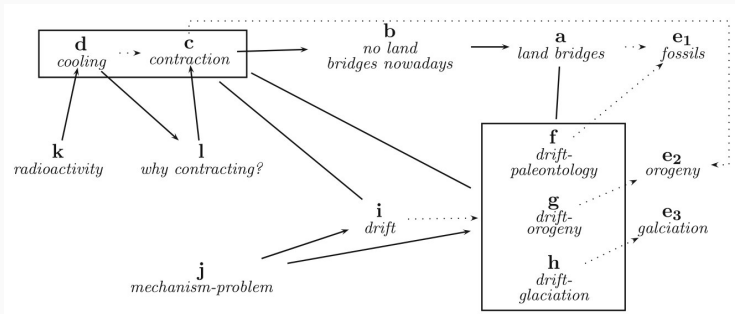
Sequent-based Argumentation: some publications

- Ofer Arieli, Christian Straßer, *Sequent-Based Logical Argumentation*, in *Argument and Computation*, Vol. 6, Issue 1, pp. 73–99, 2015
- Ofer Arieli, Christian Straßer, *Dynamic Derivations for Sequent-Based Deductive Argumentation*, Proceedings of Computational Models of Argument (Editors: S. Parsons, N. Oren, C. Reed, and F. Cerutti) in the series *Frontiers in Artificial Intelligence and Applications*, Volume 266, IOS Press, pp. 89–100, 2014
- Christian Straßer and Ofer Ariel, *Normative Reasoning by Sequent-Based Argumentation*, *Journal of Logic and Computation*, doi.org/10.1093/logcom/exv050 (2015)
- Ofer Arieli and Christian Straßer, *Deductive argumentation by enhanced sequent calculi and dynamic derivations*, *Electronic Notes in Theoretical Computer Science*, 323, 21–37 (2016).
- Ofer Arieli, Annemarie Borg, and Christian Straßer, *Argumentative Approaches to Reasoning with Consistent Subsets of Premises* in proceedings of IEA/AIE'2017 (full paper), *Lecture Notes in Artificial Intelligence series*, Springer (2017)

Agent-Based Models

- joint work: AnneMarie Borg, Daniel Frey, Dunja Šešelja and Christian Straßer
- Borg A., Frey D., Šešelja D. and Straßer C. (accepted) An Argumentative Agent-Based Model of Scientific Inquiry, forthcoming in the Proceedings of IEA/AIE, Springer-Verlag (extended version at: <https://arxiv.org/abs/1612.04432>)
- Borg A., Frey D., Šešelja D. and Straßer C. (under revision) Epistemic Effects of Scientific Interaction: approaching the question with an argumentative agent-based model, special issue of Historical Social Research: “Agent Based Modelling across Social Science, Economics, and Philosophy”

Explanatory Argumentation Frameworks



Šešelja and Straßer, *Synthese*, 2013, 190:2195–2217

Abstract argumentation in our ABM

We represent in an abstract way:

- arguments
- discovery relation
- attack relation

