Reasoning by Cases in Structured Argumentation.

Jesse Heyninck, Mathieu Beirlaen and Christian Straßer

Workgroup for Non-Monotonic Logics and Formal Argumentation
Institute for Philosophy II
Ruhr University Bochum

The 32nd ACM SIGAPP Symposium On Applied Computing
Marrakech
2017-04-5

RUHR UNIVERSITÄT BOCHUM RUB

Research Group for Non-Monotonic Logic and Formal Argumentation

1. Reasoning by Cases in Structured Argumentation
   - Structured Argumentation (without RBC)
   - Reasoning By Cases in Structured Argumentation
   - Contamination Problems

2. Comparison
   - The Meta-Rule Approach: OR
   - Disjunctive Default Logic

Reasoning by Cases

$$\dfrac{\begin{array}{c}\Gamma \\ \vdots \\ \phi \vee \psi\end{array} \quad \begin{array}{c}\Gamma, [\phi]^1 \\ \vdots \\ \delta\end{array} \quad \begin{array}{c}\Gamma, [\psi]^1 \\ \vdots \\ \delta\end{array}}{\delta} \vee E_1$$

- expressed with $\supset$:

$$\frac{\phi \vee \psi \quad \phi \supset \delta \quad \psi \supset \delta}{\delta}$$

Reasoning by Cases, Defeasibly

- strict rules ("$\rightarrow$") vs. defeasible rules ("$\Rightarrow$")
- schematically:

$$\frac{\phi \vee \psi \quad \phi \Rightarrow \delta \quad \psi \Rightarrow \delta}{\delta}$$

i Either Andrea or Bart will win the bet.
ii If Andrea wins the bet, Carolina will likely get a free drink.
iii If Bart wins the bet, Carolina will likely get a free drink.
$\therefore$ Carolina will likely get a free drink.

Reasoning by Cases, Defeasibly

- strict rules ("$\rightarrow$") vs. defeasible rules ("$\Rightarrow$")
- schematically:

$$\frac{\phi \vee \psi \quad \phi \Rightarrow \delta \quad \psi \Rightarrow \delta}{\delta}$$

- or, more generally:

$$\frac{\phi \vee \psi \quad \phi \Rightarrow \cdots \Rightarrow \delta \quad \psi \Rightarrow \cdots \Rightarrow \delta}{\delta}$$

- or, more generally:

$$\frac{\phi \vee \psi \quad \phi \mid\!\sim \delta \quad \psi \mid\!\sim \delta}{\delta}$$

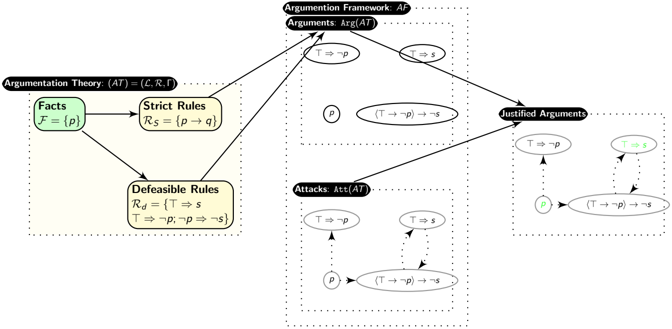
Read $\phi \mid\!\sim \delta$: "δ follows defeasibly from ϕ" or "There is a (defeasible) argument for δ based on ϕ."

Reasoning by Cases in Structured Argumentation

Structured Argumentation (without RBC)

ASPIC

- Input: argumentation theory consisting of strict and defeasible rules and factual premisses
- Argumentation theory is used to construct arguments ...
- ... that attack each other when in conflict
- Argumentation semantics are used to determine which arguments can be reasonably upheld



Argumentation Theory *AT*

Definition (Argumentation theory)

An argumentation theory (AT) is a triple $AT = (\mathcal{L}, \mathcal{R}, \mathcal{F})$ where:

- $\mathcal{L}$ is our formal language defined above;
- $\mathcal{R} = \mathcal{S} \cup \mathcal{D}$ is a set of strict ($\mathcal{S}$) and defeasible ($\mathcal{D}$) inference rules of the form $\varphi_1, \ldots, \varphi_n \rightarrow \psi$ and $\varphi_1, \ldots, \varphi_n \Rightarrow \psi$ respectively (where $\varphi_1, \ldots, \varphi_n, \psi \in \mathcal{L}$); and
- $\mathcal{F} \subseteq \mathcal{L}$ is a **CL**-consistent knowledge base.[a]

[a] $\mathcal{F} \subseteq \mathcal{L}$ is **CL**-*consistent* iff $\mathcal{F} \nvdash \bot$.

We assume in addition that $\varphi_1, \ldots, \varphi_n \rightarrow \psi \in \mathcal{S}$ iff $\{\varphi_1, \ldots, \varphi_n\} \vdash \psi$. Since we keep $\mathcal{L}$ and $\mathcal{S}$ fixed, we will in the remainder refer to ATs as pairs $(\mathcal{D}, \mathcal{F})$.

ASPIC: Arguments

- AT-arguments are proof trees based on $AT = (\mathcal{D}, \mathcal{F})$

Definition (Argument)

Where $AT = (\mathcal{L}, \mathcal{S} \cup \mathcal{D}, \mathcal{F})$, $\texttt{Arg}(AT)$ contains all A where:

(i) $A = \varphi$ if $\varphi \in \mathcal{F}$.
  - $\text{Conc}(A) = \varphi$.
  - $\text{Sub(A)} = \{\text{A}\}$.

(ii) $A = A_1, \ldots, A_n \rightarrow \psi$ where $A_1, \ldots, A_n \in \texttt{Arg}(AT)$ are such that there exists a strict rule $\text{Conc}(A_1), \ldots, \text{Conc}(A_n) \rightarrow \psi$ in $\mathcal{S}$.
  - $\text{Conc}(A) = \psi$.
  - $\text{Sub(A)} = \bigcup_{1 \leqslant i \leqslant n} \text{Sub}(\text{A}_i) \cup \{\text{A}\}$.

(iii) $A = A_1, \ldots, A_n \Rightarrow \psi$ where $A_1, \ldots, A_n \in \texttt{Arg}(AT)$ are such that there exists a defeasible rule $\text{Conc}(A_1), \ldots, \text{Conc}(A_n) \Rightarrow \psi$ in $\mathcal{D}$.
  - $\text{Conc}(A) = \psi$.
  - $\text{Sub(A)} = \bigcup_{1 \leqslant i \leqslant n} \text{Sub}(\text{A}_i) \cup \{\text{A}\}$

ASPIC: Attacks

- Idea: express that arguments contain mutually incompatible information.
- Recall: only source of defeasibility are defeasible rules

Attacks

- A attacks $(A_1, \ldots, A_n \Rightarrow \psi)$ if $\text{Conc(A)} = \neg\psi$ or $\neg\text{Conc(A)} = \psi$.
- A attacks B if A attacks B' and $B' \in \text{Sub(B)}$.

- $(A, B) \in \texttt{Att}(AT)$ iff $A, B \in \texttt{Arg}(AT)$ and A attacks B.

Argumentation Framework

- Using the arguments based on an argumentation theory AT and the attacks between them, we construct a directed graph called an **argumentation framework**.

Definition

The structured argumentation framework (in short, SAF) defined by the theory AT is the pair $(\mathtt{Arg}(AT), \mathtt{Att}(AT))$

- We then use standard Dung argumentation semantics to determine which arguments can be reasonably upheld together.

Reasoning By Cases in Structured Argumentation

A New Type of Argument: RbC-Arguments

- **Basic idea**: Given
  - an argumentation theory $AT = \langle \mathcal{D}, \mathcal{F} \rangle$
  - an argument $A_1 \in \texttt{Arg}(AT)$ for which $\text{Conc}(A_1) = \phi \vee \psi$,
  - an argument $A_2 \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{\phi\} \rangle)$ with $\text{Conc}(A_2) = \delta$, and
  - an argument $A_3 \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{\psi\} \rangle)$ with $\text{Conc}(A_3) = \delta$,
- we introduce a new RbC-Argument : $\boxed{\langle A_1, [A_2], [A_3] \rightsquigarrow \delta \rangle}$.

More general: RbC-Argument

Definition

Where

- $A_0 \in \mathtt{Arg}(AT)$ with $\mathsf{Conc}(A_0) = \bigvee_{i=1}^{n} \phi_i$ and
- $A_i \in \mathtt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{\phi_i\}\rangle) \setminus \mathtt{Arg}(AT)$ $(1 \leq i \leq n)$,

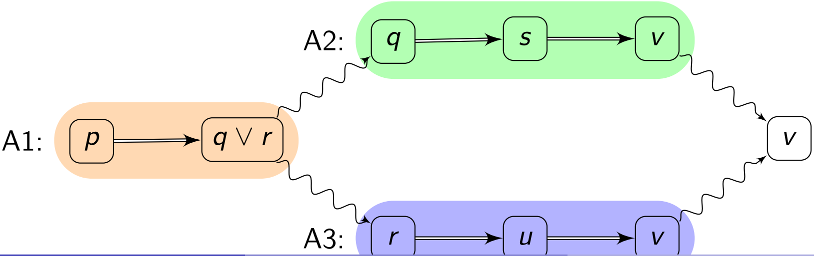
$A = \langle A_0, [A_1], \ldots, [A_n] \rightsquigarrow \bigvee_{i=1}^{n} \mathsf{Conc}(A_i)\rangle$ is an RbC-argument.

- $\mathsf{Sub}(A) = \{A\} \cup \mathsf{Sub}(A_1)$.
- $\mathsf{Conc}(A) = \mathsf{Conc}(A_i)$

- We say that $A_1, \ldots, A_n$ are **hypothetical sub-arguments** of A based on respective the hypotheses $\phi_1, \ldots, \phi_n$,
  in signs: $(A_1, \phi_1), \ldots, (A_n, \phi_n) \in \mathsf{HSub}(A)$.
- $\mathtt{HArg}(AT) =_{\mathsf{df}} \{A \mid \langle A, \phi\rangle \in \mathsf{HSub}(A), A \in \mathtt{Arg}(AT)\}$

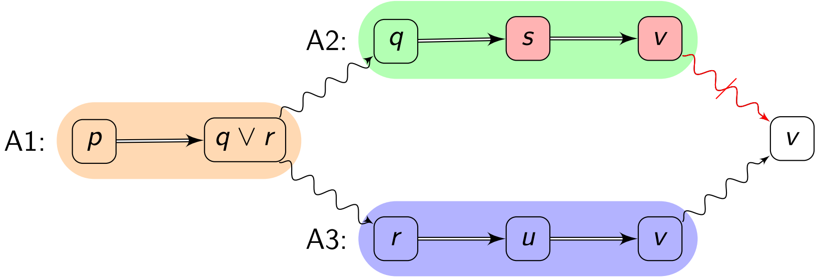
Example

- Let $AT = \langle \mathcal{D}, \mathcal{F} \rangle$ consist of $\mathcal{D} = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, t \Rightarrow \neg s\}$ and $\mathcal{F} = \{p, t\}$.
- We have for instance the arguments:
- $A_1 = \langle \langle p \rangle \Rightarrow q \vee r \rangle \in \texttt{Arg}(AT)$
- $A_2 = \langle \langle q \rangle \Rightarrow s \Rightarrow v \rangle \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{q\} \rangle)$
- $A_3 = \langle \langle r \rangle \Rightarrow u \Rightarrow v \rangle \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{K} \cup \{r\} \rangle)$
- $A_4 = \langle A_1, [A_2], [A_3] \rightsquigarrow v \rangle \in \texttt{Arg}(AT)$.



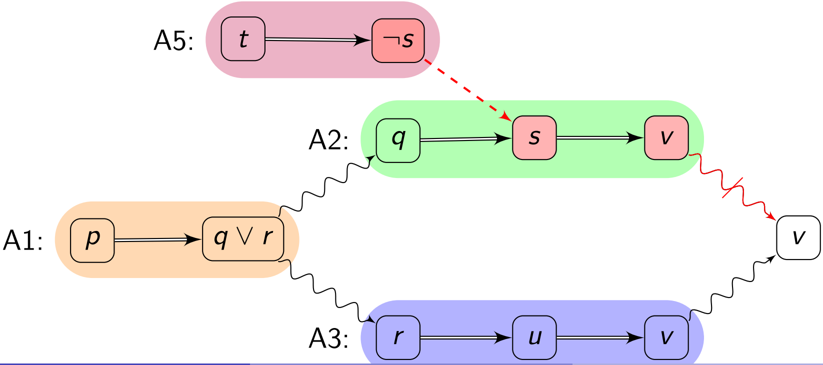
Attacks

- $A_1 = \langle\langle p\rangle \Rightarrow q \vee r\rangle \in \texttt{Arg}(AT)$
- $A_2 = \langle\langle q\rangle \Rightarrow s \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{q\}\rangle)$
- $A_3 = \langle\langle r\rangle \Rightarrow u \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{r\}\rangle)$
- $A_4 = \langle A_1, [A_2], [A_3] \rightsquigarrow v\rangle \in \texttt{Arg}(AT)$



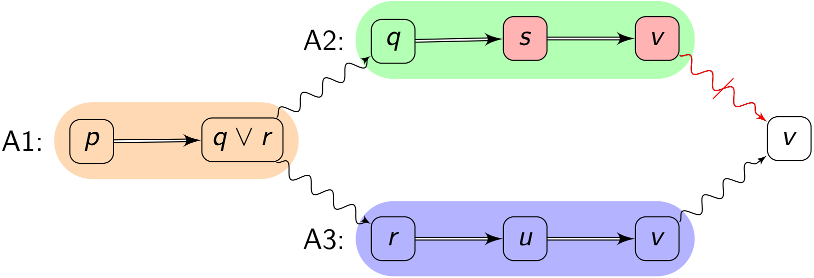
Attacks

- $A_1 = \langle\langle p\rangle \Rightarrow q \vee r\rangle \in \texttt{Arg}(AT)$
- $A_2 = \langle\langle q\rangle \Rightarrow s \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{q\}\rangle)$
- $A_3 = \langle\langle r\rangle \Rightarrow u \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{r\}\rangle)$
- $A_4 = \langle A_1, [A_2], [A_3] \rightsquigarrow v\rangle \in \texttt{Arg}(AT)$
- $A_5 = \langle\langle t\rangle \Rightarrow \neg s\rangle \in \texttt{Arg}(AT)$



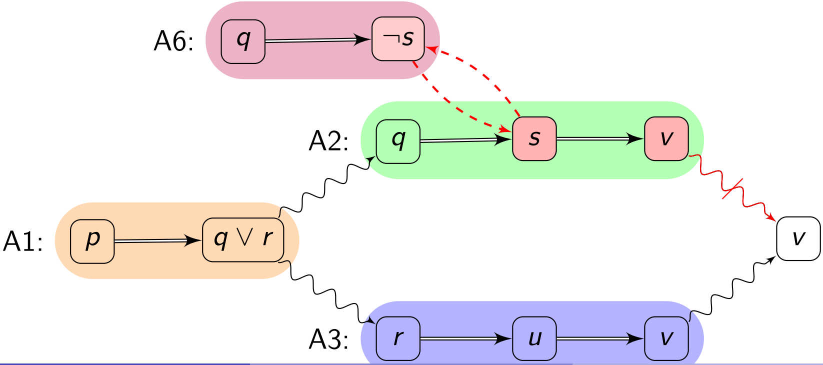
Attacks

- $A_1 = \langle\langle p\rangle \Rightarrow q \vee r\rangle \in \texttt{Arg}(AT)$
- $A_2 = \langle\langle q\rangle \Rightarrow s \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{q\}\rangle)$
- $A_3 = \langle\langle r\rangle \Rightarrow u \Rightarrow v\rangle \in \texttt{Arg}(\langle\mathcal{D}, \mathcal{F} \cup \{r\}\rangle)$
- $A_4 = \langle A_1, [A_2], [A_3] \rightsquigarrow v\rangle \in \texttt{Arg}(AT)$



Attacks

- $A_1 = \langle\langle p\rangle \Rightarrow q \vee r\rangle \in \texttt{Arg}(AT)$
- $A_2 = \langle\langle q\rangle \Rightarrow s \Rightarrow v\rangle \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{q\}\rangle)$
- $A_3 = \langle\langle r\rangle \Rightarrow u \Rightarrow v\rangle \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{r\}\rangle)$
- $A_4 = \langle A_1, [A_2], [A_3] \rightsquigarrow v\rangle \in \texttt{Arg}(AT)$
- $A_6 = \langle\langle q\rangle \Rightarrow \neg s\rangle \in \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{q\}\rangle)$



Attacks: the Full Definition

Definition

$\texttt{Arg}(AT^{\phi}) =_{\text{df}} \texttt{Arg}(\langle \mathcal{D}, \mathcal{F} \cup \{\phi\}\rangle) \setminus \texttt{Arg}(AT)$

Definition

$\texttt{Att} \subseteq \big(\texttt{Arg}(AT) \times (\texttt{HArg}(AT)) \cup \texttt{Arg}(AT)\big) \cup \big(\texttt{HArg}(AT) \times \texttt{HArg}(AT)\big)$
where $(A; B = B_1, \ldots, B_n \Rightarrow \phi) \in \texttt{Att}$ iff

- $\text{Conc(A)} = \neg\phi$ or $\neg\text{Conc(A)} = \phi$ and:
  - $A \in \texttt{Arg}(AT)$ and $B \in (\texttt{HArg}(AT)) \cup \texttt{Arg}(AT)$ or
  - $A \in \texttt{Arg}(AT^{\psi})$ and $B \in \texttt{Arg}(AT^{\psi})(AT)$ (for any $\psi \in \mathcal{L}$).

Definition

Lifting for Sub-Arguments A attacks B if there is an $B' \in \text{Sub(B)} \cup \{\text{C}' \mid \text{C}' \in \text{Sub(C)}, \langle \text{C}, \phi\rangle \in \text{HSub(B)}\}$ s.t. A attacks B'.

Argumentation Frameworks for RBC-args

Definition

The structured argumentation framework (in short, SAF) defined by the theory AT is the pair $(\texttt{Arg}(AT) \cup \texttt{HArg}(AT), \texttt{Att}(AT))$

Argumentation Semantics

Definition (Defense)

A set of arguments $\mathcal{S} \subseteq \texttt{Arg}(AT) \cup \texttt{HArg}(AT)$ *defends an argument* A *iff every attacker of* A *is attacked by some* $B \in \mathcal{S}$.

Definition (Conflict-free)

A set of arguments $\mathcal{S} \subseteq \texttt{Arg}(AT) \cup \texttt{HArg}(AT)$ is *conflict-free* if there are no $A, B \in \mathcal{S}$ for which $(A, B) \in \texttt{Att}(AT)$.

Extensions

Definition (Extensions)

Let $(\texttt{Arg}(AT) \cup \texttt{HArg}(AT), \texttt{Att}(AT))$ be a SAF. If $\mathcal{S} \subseteq \texttt{Arg}(AT) \cup \texttt{HArg}(AT)$ is conflict-free, then:

- $\mathcal{S}$ is a *complete* extension iff $A \in \mathcal{S}$ whenever $\mathcal{S}$ defends A;
- $\mathcal{S}$ is the *grounded* extension iff it is the set inclusion minimal complete extension.
- $\mathcal{S}$ is a *preferred* extension iff it is a set inclusion maximal complete extension.
- $\mathcal{S}$ is the *stable* extension iff it attacks every $A \in \texttt{Arg}(AT) \cup \texttt{HArg}(AT) \setminus \mathcal{S}$.

Consequences

Definition

Let $\mathsf{SAF} = (\mathtt{Arg}(\mathcal{AT}) \cup \mathtt{HArg}(\mathcal{AT}), \mathtt{Att}(\mathcal{AT}))$, let $\mathsf{sem} \in \{\mathsf{Cmp}, \mathsf{Prf}, \mathsf{Grd}\}$, and let Cmp(SAF), Prf(SAF), and Grd(SAF) denote the sets of SAF's complete extensions, SAF's preferred extensions, and SAF's grounded extension respectively.

- $\mathsf{SAF} \mid\!\sim^{\cap}_{\mathsf{sem}} \phi$ iff for every $\mathcal{B} \in \mathsf{sem}(\mathsf{SAF})$ there is an $A \in \mathcal{B} \cap \mathtt{Arg}(\mathcal{AT})$ with $\mathrm{conc}(A) = \phi$.
- $\mathsf{SAF} \mid\!\sim^{\Cap}_{\mathsf{sem}} \phi$ iff there is a $A \in \bigcap \mathsf{sem}(\mathsf{SAF}) \cap \mathtt{Arg}(\mathcal{AT})$ with $\mathrm{conc}(A) = \phi$.

Since the grounded extension is unique both definitions coincide for $\mathsf{sem} = \mathsf{Grd}$.

Rationality Postulates [3, 4]

Theorem (Direct Consistency)

Let $(\mathrm{HArg}(AT), \mathtt{Att}(AT))$ *be a SAF and* $\mathcal{A}$ *an extension using any semantics subsumed by complete semantics. Then there are no* $A, B \in \mathcal{A} \cap \mathtt{Arg}(AT)$ *s.t.* $\mathrm{Conc(A)} = -\mathrm{Conc(B)}$.

Theorem (Closure)

Let $(\mathtt{Arg}(AT), \mathtt{Att}(AT))$ *be a SAF and* $\mathcal{A}$ *an extension using any semantics subsumed by complete semantics. If* $A_1, \ldots, A_n \in \mathcal{A} \cap \mathtt{Arg}(AT)$ *and* $\bigcup_{1 \leqslant i \leqslant n} \mathrm{Conc(A_i)} \vdash \phi$ *then* $A_1, \ldots, A_n \rightarrow \phi \in \mathcal{A} \cap \mathtt{Arg}(AT)$.

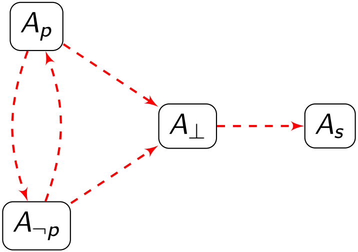
Theorem (Indirect Consistency)

Let $(\mathtt{Arg}(AT), \mathtt{Att}(AT))$ *be a SAF and* $\mathcal{A}$ *an extension using any semantics subsumed by complete semantics. Then there are no* $A_1, \ldots, A_n \in \mathcal{A} \cap \mathtt{Arg}(AT)$ *s.t.* $\bigcup_{1 \leqslant i \leqslant n} \{\mathrm{Conc(A_i)}\} \vdash \bot$.

Contamination Problems

The Contaminating Power of Inconsistent Arguments

- Suppose $\mathcal{D} = \{\Rightarrow p, \Rightarrow \neg p, \Rightarrow s\}$ and $\mathcal{F} = \emptyset$
- In this case we clearly want s to be a consequence.
- Let's see what happens. We have, for instance, the arguments:
  - $A_p = \langle \Rightarrow p \rangle$
  - $A_{\neg p} = \langle \Rightarrow \neg p \rangle$
  - $A_s = \langle \Rightarrow s \rangle$
  - $A_\perp = \langle A_p, A_{\neg p} \rightarrow \neg s \rangle$



Filtering Out Inconsistent Arguments

Definition

Where $a \in \text{Arg}(T)$ we define $\dagger(a)$ as inductively follows:

- $\dagger(\langle \phi \rangle) = \phi$
- $\dagger(\langle A_1, \ldots, A_n \to \phi \rangle) = \dagger(A_1) \wedge \cdots \wedge \dagger(A_n)$
- $\dagger(\langle A_1, \ldots, A_n \Rightarrow \phi \rangle) = \dagger(A_1) \wedge \cdots \wedge \dagger(A_n) \wedge \phi$
- $\dagger(\langle A_1, [A_2], \ldots, [A_n] \rightsquigarrow \phi \rangle) = \dagger(A_1) \wedge (\dagger(A_2) \vee \cdots \vee \dagger(A_n))$

Definition

An argument $A \in \text{Arg}(T)$ is *inconsistent* iff $\mathcal{K} \vdash \neg \dagger (A)$.

Comparison

The Meta-Rule Approach: OR

- Rules for rules:

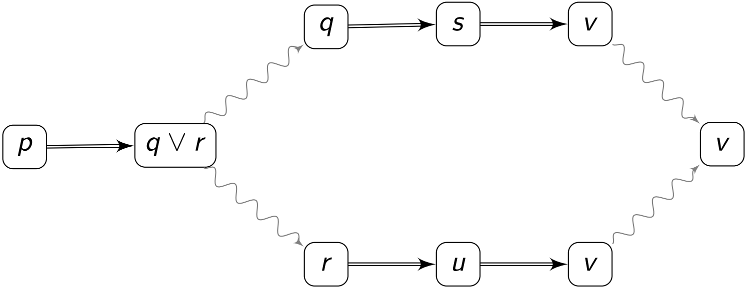
$$\frac{\phi \Rightarrow \delta \quad \psi \Rightarrow \delta}{\phi \vee \psi \Rightarrow \delta} \text{ [OR]}$$

- Illustration:

1.	$\phi \Rightarrow \delta$	PREM
2.	$\psi \Rightarrow \delta$	PREM
3.	$\phi \vee \psi$	PREM
4.	$\phi \vee \psi \Rightarrow \delta$	1,2; OR
5.	δ	3,4; DefeasibleMP

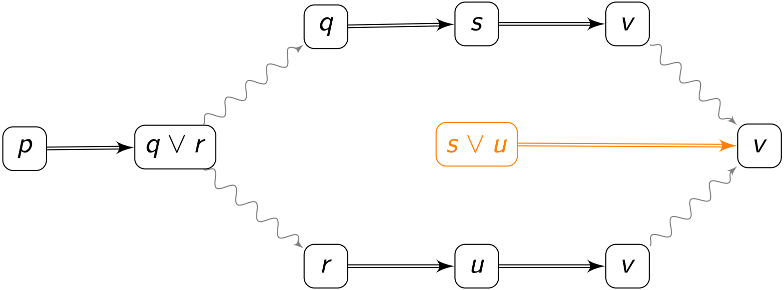
OR: example

- Suppose we have $\Sigma = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p\}$.



OR: example

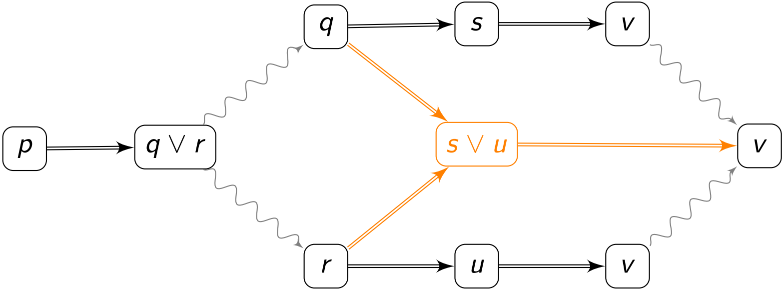
- Suppose we have $\Sigma = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p\}$.



- by **(OR)**: from $s \Rightarrow v$ and $u \Rightarrow v$

OR: example

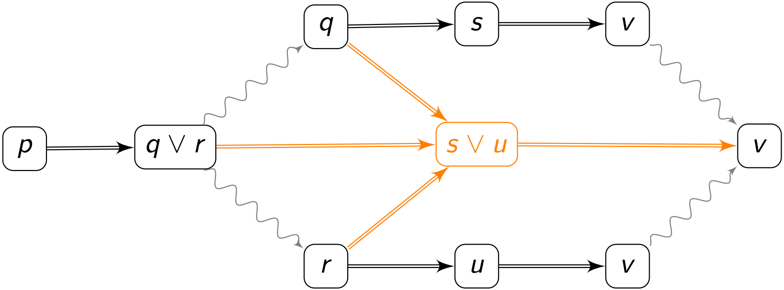
- Suppose we have $\Sigma = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p\}$.



- by **(OR)**: from $s \Rightarrow v$ and $u \Rightarrow v$
- by **(Right-Weakening)**, from $q \Rightarrow s$ and $r \Rightarrow u$

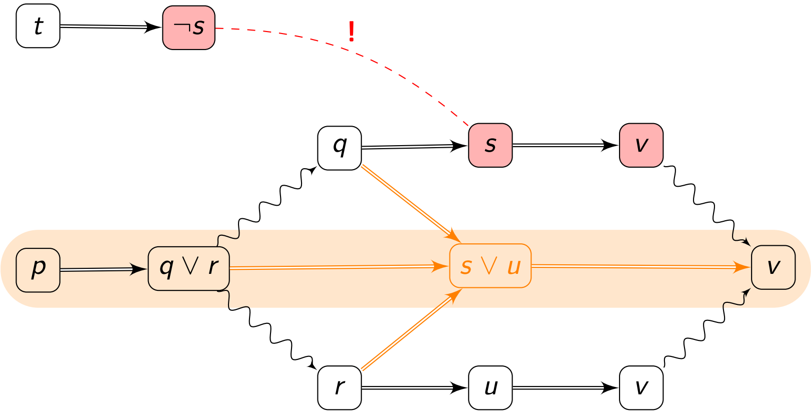
OR: example

- Suppose we have $\Sigma = \{p \Rightarrow q \vee r, q \Rightarrow s, s \Rightarrow v, r \Rightarrow u, u \Rightarrow v, p\}$.



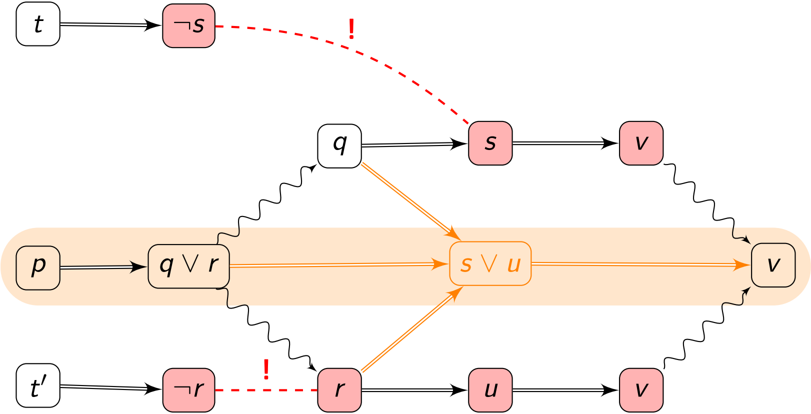
- by **(OR)**: from $s \Rightarrow v$ and $u \Rightarrow v$
- by **(Right-Weakening)**, from $q \Rightarrow s$ and $r \Rightarrow u$
- by **(OR)**: from $q \Rightarrow s \vee u$ and $r \Rightarrow s \vee u$

A Problematic Example for OR



- Suppose now we also have t and $t \Rightarrow \neg s$.
- the possible defeater has no effect on the generalized path

A Problematic Example for OR

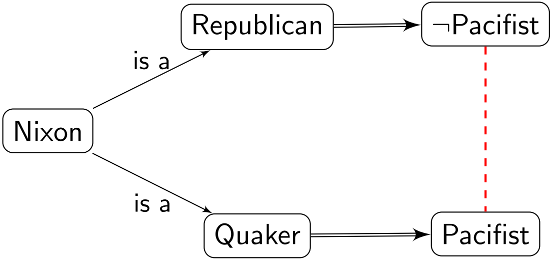


- Suppose now we also have t' and $t' \Rightarrow \neg r$.
- the additional possible defeater has no effect on the generalized path

Disjunctive Default Logic

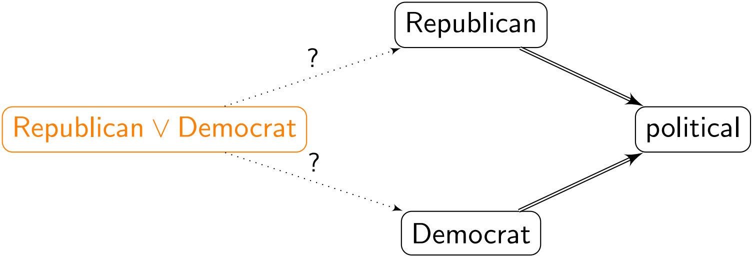
Default Logic

- Input: a set of defaults and a set of formulas ("facts")
- Build extensions by applying Modus Ponens to defaults while maintaining consistency



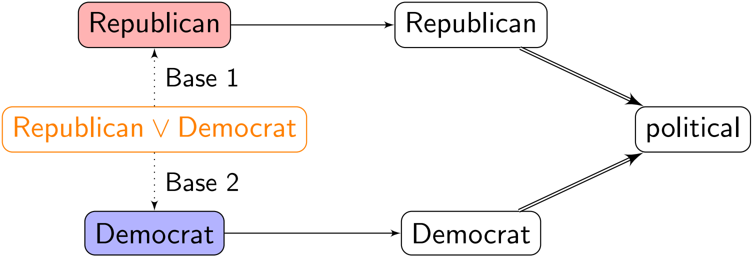
Default Logic: Disjunctive Information

- No machinery for handling disjunctive information in original Reiter paper.
- for instance: $\Sigma = \{\text{Republican} \vee \text{Democrat}, \text{Republican} \Rightarrow \text{political}, \text{Democrat} \Rightarrow \text{political}\}$.



Disjunctive Default Logic [5]

- Split disjunctive information into disjuncts
- So a set of facts gets split into two bases for every disjunction it contains
- for instance: $\Sigma = \{\text{Republican} \vee \text{Democrat}, \text{Republican} \Rightarrow \text{political}, \text{Democrat} \Rightarrow \text{political}\}$.



Two extensions:

- Republican, political
- Democrat, political

Problematic Example for Disjunctive Default Logic

- Consider the following example:

1. Either his left hand or his right hand is broken. $\boxed{lhb \vee rhb}$
2. If somebody writes legibly then usually the right hand is not broken. $\boxed{wl \Rightarrow \neg rhb}$
3. He writes legibly. $\boxed{wl}$

- With disjunctive default logic we get two extensions:

  $wl, \neg rhb, lhb$

  wl, rhb

Existing Approaches: **Manipulate the database!**

1. produce new defeasible rules from the given ones
2. produce new factual knowledge bases when confronted with disjunctive information

Enters: **the Argumentative Approach**

- Instead of manipulating the knowledge base and reasoning on top of the manipulated database,
- in our approaches we have a **more direct approach** to the modeling of Reasoning by Cases in the context of defeasible reasoning, following the inference scheme:

$$\frac{\phi \vee \psi \quad \phi \Rightarrow \cdots \Rightarrow \delta \quad \psi \Rightarrow \cdots \Rightarrow \delta}{\delta}$$

or, more generally:

$$\frac{\phi \vee \psi \quad \phi \mathrel{|\!\sim} \delta \quad \psi \mathrel{|\!\sim} \delta}{\delta}$$

- This allows us to have more control over defeating conditions ...
- ... and to avoid pitfalls as the ones demonstrated above.

Thanks for your attention.

Questions?

Bibliography I

D. A. Bochvar.

On a three-valued calculus and its application to analysis of paradoxes of classical extended functional calculus.

Matématičéskij Sbnornik, 4:287–308, 1938.

M. Caminada.

Dialogues and HY-arguments.

In J. Delgrande and T. Schaub, editors, *10th International Workshop on Non-Monotonic Reasoning (NMR 2004), Whistler, Canada, June 6-8, 2004, Proceedings*, pages 94–99, 2004.

M. Caminada and L. Amgoud.

On the evaluation of argumentation formalisms.

Artificial Intelligence, 171:286 – 310, 2007.

Bibliography II

M. Caminada, W. Carnielli, and P. Dunne.
Semi-stable semantics.
Journal of Logic and Computation, 22(5):1207–1254, 2012.

M. Gelfond, V. Lifschitz, H. Przymusinska, and M. Truszczynski.
Disjunctive defaults.
In *Proceedings of International Conference on Principles of Knowledge Representation and Reasoning (KR)*, pages 230–237, 1991.

G. Malinowski.
Many-valued logic and its philosophy.
In D. Gabbay and J. Woods, editors, *The Many Valued and Nonmonotonic Turn in Logic*, volume 8 of *Handbook of the History of Logic*, pages 13–94. Elsevier, 2007.

Bibliography III

S. Modgil and H. Prakken.
The ASPIC+ framework for structured argumentation: a tutorial.
Argument & Computation, 5(1):31–62, 2014.