THE ELLSBERRY PARADOX AND THE WEIGHT OF ARGUMENTS

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**Standard Approach: Maximize Expected Utility**

- Expected utility of an action: the sum of the products of multiplying (1) the probability of each circumstance given an action by (2) the utility for that action.

- Maximize expected utility: act so that expected utility is as great as possible.

- If expected utilities of actions are equal, then you should be indifferent.
THE ELLSBERT PARADOX

- Paradox for MEU.

- There is a box with-
  
  1/3 black balls

  Between 0 and 2/3 green balls

  Between 0 and 2/3 red balls

- There are two choices between bets on a randomly selected ball from the box.
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<th>BLACK</th>
<th>GREEN</th>
<th>RED</th>
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<tbody>
<tr>
<td><strong>A</strong></td>
<td>£100</td>
<td>£0</td>
<td>£0</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td>£0</td>
<td>£100</td>
<td>£0</td>
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</table>

**A:** “The ball will be black.”  
**B:** “The ball will be green.”

In experiments, most people prefer A to B.
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<tr>
<td><strong>C</strong></td>
<td>£100</td>
<td>£0</td>
<td>£100</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>£0</td>
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**C**: “The ball will not be green.”  
**D**: “The ball will not be black.”

In experiments, most people prefer D to C.
THE PARADOX

- The EU of betting A is greater than the EU of B iff the EU of C is greater than the EU of D.

- Why A > B?
  - Only one possible reason in MEU theory: more likely that the ball will be red rather than green.

- But then why not C > D?

- MEU: combination is irrational
PROBLEM

Nothing formally wrong or intuitively irrational.

Expected utilities CAN be equal.

Conservative solution?
EVIDENTIAL PROBABILITY

- Developed by Henry E. Kyburg (1928-2007)
- Provides a system whereby all probabilities are derived from information about relative frequencies.
- Single probability for given evidence.
- Evidential probabilities can be imprecise.
- When information is imprecise.
SPECULATION AND DECISION

• How do we get a decision-theory with Evidential Probabilities?

• Speculate relative frequency information that is consistent with the Evidential Probabilities.

• Bet as if we knew the relative frequencies.
EXAMPLE

- Tossing Gömböc: very imprecise prob.
  - Maybe [0, 1]

- Tossing a 1 euro coin: relatively precise prob.
  - Like [0.49, 0.51]

- Many would speculate: 0.5 (1/2)
SPECULATION AND DECISION

There is a pre-theoretical distinction between-

(1) Making decisions based on evidence.

(2) Making decisions based on speculation.

A difference of degrees – measure with Evidential Probabilities.

A tie-breaker *if* expected utilities are equal.
IMPRECISION AS A DECISION TOOL

- Bet with even odds.

- Gömböc or coin?

- Coin, because less speculation.
THE ELLSBERG PARADOX

You know that $1/3$ balls are black and that $[0, 2/3]$ are green.

You might speculate that $1/3$ are green.

EU for each choice is equal.

A is less speculative than B.

D is less speculative than C.
WEIGHT OF ARGUMENTS

- John Maynard Keynes: quantity of relevant evidence (in an argument for some action) matters.

- But how?

- It can help us choose when expected utilities are equal.
CONCLUSIONS

A conservative response to the Ellsberg Paradox?

- Yes.

Is Evidential Probability AND precise decision theory?

- Yes.

Does the Weight of Argument matter? Sometimes.