Argument Evaluation Based on Proportionality

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Argument Strength
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The strength of structured arguments

I. Syntax
II. Evaluation
III. Attack relation
I. Syntax
Classical diagrams of arguments

Simple argument

Linked argument

Serial argument

Convergent argument

Divergent argument

Multilevel complex argument
Conductive arguments (i.e. pro-contra, cf. Walton & Gordon 2015)

Conductive argument

Con-argument

Problematic cases
Multilevel convergent argument (an example)

- premise
- conclusion
- first premise
- intermediate conclusion
- final conclusion, final argument
- atomic argument (= either simple or linked)
Formal structure of arguments

Two kinds of inference:

• pro-premises support conclusions;
• con-premises deny (contradict) conclusions.

Formal representation (Selinger 2014, 2015):

• Let $L$ be a language, i.e. a set of sentences.

  Sequent is any tuple of the form $<P, c, d>$, where:
  • $P$ is a finite, non-empty set of sentences of $L$ (premises);
  • $c$ is a single sentence of $L$ (conclusion);
  • $d$ is a Boolean value (1 in pro-sequents and 0 in con-sequents).

  Argumentation structures (arguments) are any finite and non-empty sets of sequents.
Atypical argumentation structures:

- arguments can have less or more than one final conclusion
- in what follows arguments will be assumed to be coherent, non-divergent and non-circular
II. Numerical evaluation
Evaluation. Formal preliminaries

- We assume that $L$ contains the negation and the conjunction connectives;
- $v: L' \rightarrow [0, 1]$, where $L' \subseteq L$, is evaluation function — $v(\alpha)$ is (the degree of) acceptability of $\alpha$;
- $w: L \times L \rightarrow [0, 1]$ is conditional acceptability — $w(\alpha/\beta)$ is the acceptability of $\alpha$ under the condition that $v(\beta) = 1$;

Question: should $w$ be a partial function?

- $v(\neg \alpha) = 1 - v(\alpha)$ — postulate of rationality;
- If some premises deny $\alpha$ then the evaluation of $\alpha$ is based on the evaluation of $\neg \alpha$ in the corresponding sequent, in which these premises support $\neg \alpha$. 
Evaluation as transforming of acceptability values

- the evaluation function is defined for the first premises
- the acceptability of the first premises is transformed step by step to the acceptability of the final conclusion
- formally, in each step the domain of the evaluation function is extended to the set containing new conclusion
The principle of proportionality

The strength of argument should vary proportionally to the values assigned to its components.
Evaluation of premises

\( x, y \) are the acceptability values of some two premises

\[
\frac{(x \cdot y)}{x} = \frac{y}{1}
\]

\( x \cdot y = xy \)
Evaluation of atomic arguments

$x$ is the acceptability of (the set) of premises;
y is the conditional acceptability (conclusion/premises)
$x, y > \frac{1}{2}$

\[
\frac{(x \otimes y)}{y} = \frac{x}{1}
\]

\[
x \otimes y = xy
\]

\[
\frac{(x \otimes' y) - \frac{1}{2}}{x - \frac{1}{2}} = \frac{y - \frac{1}{2}}{\frac{1}{2}}
\]

\[
x \otimes' y = 2xy - x - y + 1
\]
Evaluation of convergent pro-arguments

\( x, y \) – acceptability values of two converging arguments

\( x, y > \frac{1}{2} \)

\[
\frac{(x \oplus y) - x}{1 - x} = \frac{y - \frac{1}{2}}{\frac{1}{2}}
\]

\( x \oplus y = 2x + 2y - 2xy - 1 \)

Yanal’s algorithm (1991)

\[
\frac{(x \oplus' y) - x}{1 - x} = \frac{y}{1}
\]

\( x \oplus' y = x + y - xy \)
Evaluation of convergent \textit{con}-arguments

\[ x, y < \frac{1}{2} \]

\[ \frac{(x \oplus_c y)}{x} = \frac{y}{\frac{1}{2}} \]

\[ x \oplus_c y = 2xy \]

\[ x \oplus_c y = 1 - [(1-x) \oplus (1-y)] = 2xy \]
Evaluation of conductive arguments

$x < \frac{1}{2}$ is the acceptability of all convergent con-arguments

$y > \frac{1}{2}$ is the acceptability of all convergent pro-arguments

\[ y \otimes x = (y - \frac{1}{2}) - (\frac{1}{2} - x) + \frac{1}{2} \]

\[ y \otimes x = y + x - \frac{1}{2} \]

\[ y - (y \otimes' x) = (y \otimes' x) - x \]

\[ y \otimes' x = \frac{1}{2} (y + x) \]
Evaluation of atomic arguments

Let \( \land A \) be the conjunction of all the propositions belonging to a finite set \( A \).

Let \( A = \{<P, c, d>\} \), where \( P \subseteq \text{dom}(v) \), \( c \notin \text{dom}(v) \), and \( d \) is a Boolean value.

The function \( v_A \) is the following extension of \( v \) to the set \( \text{dom}(v) \cup \{c\} \):

- If \( d = 1 \) then \( v_A(c) = v(\land P) \cdot w(c/\land P) \);
- If \( d = 0 \) then \( v_A(c) = 1 - v(\land P) \cdot w(\neg c/\land P) \).

The value \( v_A(c) \) can be called the (logical) strength or force of the argument \( A \).

Note: the strength of \( \{<P, c, 0>\} = 1 - \) the strength of \( \{<A, \neg c, 1>\} \).

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**Acceptability** of arguments:

- If \( A \) is a *pro*-argument then it is acceptable iff \( v_A(c) > \frac{1}{2} \);
- If \( A \) is a *con*-argument then it is acceptable iff \( v_A(c) < \frac{1}{2} \).
Evaluation of convergent arguments

Let $A$ be an argument, $\alpha$ its final conclusion, and let $A = A_1 \cup A_2$, where both $A_1$ and $A_2$ have the same final conclusion $c$, they are coherent and acceptable, and all their sequents whose conclusion is $c$ are only either pro- or con-sequents.

- If both $A_1$ and $A_2$ are pro, and $\nu_{A_1}(c) > \frac{1}{2}$, then $\nu_A(c) = \nu_{A_1}(c) \oplus \nu_{A_2}(c)$
- If both $A_1$ and $A_2$ are con, and $\nu_{A_1}(c) < \frac{1}{2}$, then $\nu_A(c) = \nu_{A_1}(c) \oplus_c \nu_{A_2}(c) = 1 - (1 - \nu_{A_1}(c)) \oplus (1 - \nu_{A_2}(c))$

where

\[ x \oplus y = 2x + 2y - 2xy - 1 \]
\[ x \oplus_c y = 2xy. \]

Note: The operations $\oplus$ and $\oplus_c$ are both commutative and associative, therefore the strengths of any number of convergent arguments can be added in any order.
Evaluation of conductive arguments

Let $A$ be an argument, $\alpha$ its final conclusion, and let $A = A_{pro} \cup A_{con}$, where all the sequents of $A_{pro}$ whose conclusion is $c$ are only $pro$-sequents and all the sequents of $A_{con}$ whose conclusion is $c$ are only $con$-sequents.

We assume that both $A_{pro}$ and $A_{con}$ are coherent and acceptable, i.e. $v_{A_{pro}}(c) > \frac{1}{2}$ and $v_{A_{con}}(c) < \frac{1}{2}$.

- If $v_{A_{pro}}(c) < 1$, and $v_{A_{con}} > 0$, then $v_A(c) = v_{A_{pro}}(c) \ominus v_{A_{con}}(c)$ where $y \ominus x = y + x - \frac{1}{2}$;

- If $v_{A_{pro}}(c) = 1$, and $v_{A_{con}}(c) \neq 0$, then $v_A(c) = 1$;

- If $v_{A_{pro}}(c) \neq 1$, and $v_{A_{con}}(c) = 0$, then $v_A(c) = 0$;

- If $v_{A_{pro}}(c) = 1$, and $v_{A_{con}}(c) = 0$, then $v_A(c)$ is not computable.
III. Attack relation (elementary cases)
Attack relation between arguments.
Rebuttals, underminers, undercutters (Prakken 2010)

• Attack on argument conclusion:

\[ \langle P_1, c, d \rangle \] can attack (rebut) \[ \langle P_2, c, 1 - d \rangle \]
\[ \langle P_1, c, d \rangle \] can attack (rebut) \[ \langle P_2, c', d \rangle \] , where \( c' = \neg c \) or \( \neg c' = c \)

• Attack on argument premises:

\[ \langle P_1, c_1, 0 \rangle \] can attack (undermine) \[ \langle P_2, c_2, d \rangle \] if \( c_1 \in P_2 \)
\[ \langle P_1, c_1, 1 \rangle \] can attack (undermine) \[ \langle P_2, c_2, d \rangle \] if \( c_1' \in P_2 \), where \( c_1' = \neg c_1 \) or \( \neg c_1' = c_1 \)

• Attack on the relationship between argument premises and argument conclusion:

undercutting defeaters
Successful attack on conclusion. Rebuttals

An argument $A$ rebuts (the conclusion of) an argument $B$ iff

- $A = \{ <P_1, c, d> \}$,
- $B = \{ <P_2, c, 1 - d> \}$
- either $d = 0$ and $1 - v_A(c) \geq v_B(c)$, or $d = 1$ and $1 - v_A(c) \leq v_B(c)$

or

- $A = \{ <P_1, c, d> \}$,
- $B = \{ <P_2, c', d> \}$, where $(c' = \neg c$ or $c = \neg c')$
- either $d = 0$ and $v_A(c) \leq v_B(c')$, or $d = 1$ and $v_A(c) \geq v_B(c')$.

Note: in the borderline cases, i.e. if the above values are equal, the conclusion is not rebutted, but it is merely questioned.
Successful attack on premises. Underminers

An argument \( A \) undermines (a premise of) an argument \( B \) iff

- \( A = \{<P_1, c_1, 0>\} \),
- \( B = \{<P_2, c_2, d>\} \), where \( c_1 \in P_2 \) is the attacked premise,
- either \( d = 1 \) and \( v'_B(c_2) = [v(c_1) \otimes v'_A(c_1)] \cdot v(\land P_2 - c_1) \cdot w(c_2/\land P_2) \leq \frac{1}{2} \), or \( d = 0 \) and \( v'_B(c_2) = [v(c_1) \otimes v'_A(c_1)] \cdot v(\land P_2 - \{c_1\}) \cdot w(\neg c_2/\land P_2) \leq \frac{1}{2} \), where \( v' \) is the function obtained from \( v \) by deleting \( c_1 \) from its domain;

or

- \( A = \{<P_1, c_1, 1>\} \),
- \( B = \{<P_2, c_2, d>\} \), where \( c_1' \in P_2 \) is attacked (\( c_1' = \neg c_1 \) or \( c_1 = \neg c_1' \)),
- either \( d = 1 \) and \( v'_B(c_2) = [v(c_1) \otimes (1 - v'_A(c_1))] \cdot v(\land P_2 - \{c_1\}) \cdot w(c_2/\land P_2) \leq \frac{1}{2} \), or \( d = 0 \) and \( v'_B(c_2) = [v(c_1) \otimes (1 - v'_A(c_1))] \cdot v(\land P_2 - \{c_1\}) \cdot w(\neg c_2/\land P_2) \leq \frac{1}{2} \), where \( v' \) is the function obtained from \( v \) by deleting \( c_1 \) and \( c_1' \) from its domain.
Undercutters. Formal representation

Pollock’s example of undercutting defeater (1987):

*The object looks red, thus it is red, unless it is illuminated by a red light.*

1) \(<P, c, d, R>\), where \(R\) is a set of (linked) rebuttals.

If \(R\) is non-empty then:

\(<P, c, d, R>\) can undercut \(<P, c, d, \emptyset>\)

2a) \(<R, \{<P, c, d>\} \text{ is not acceptable, 1}>\)
2b) \(<R, \{<P, c, d>\} \text{ is acceptable, 0}>\)

2a) or 2b) can undercut \(<P, c, d>\)
Undercutters. Formal representation

Changing the categorial classification of the attack relation, i.e. including (sets of) sentences to its domain:

\[ R \text{ can attack (undercut)} \{<P, c, d>\} \]

The sentence ‘the object X is illuminated by a red light’ can undercut the argument ‘the object X looks red, thus it is red’.
Relevance of undercutters. Hybrid arguments (Vorobej 1995)

The object looks red.  

The object is not illuminated by the red light.  

The object is red.

The relevance condition for undercutters:

\[ w(c/\wedge P \wedge \neg \wedge R) > w(c/\wedge P) \]
Undercutters. Evaluation

$x$ is the conditional acceptability of an attacked argument (conclusion/premises); $y$ is the acceptability of its undercutter; $x, y > \frac{1}{2}$

\[
\frac{(x \circ y) - \frac{1}{2}}{x - \frac{1}{2}} = \frac{1 - y}{\frac{1}{2}}
\]

\[
x \circ y = 2x + y - 2xy - \frac{1}{2}
\]
Successful attack on the relationship between premises and conclusion. Undercutters

A set of sentences $R$ *undercuts* an argument $B$ iff

- $B = \{<P, c, 1>\}$,
- $R$ is a relevant undercutter for $B$, i.e. $w(c/P\wedge\neg\neg R) > w(c/P)$,
- $v_{B,R}(c) = v(\neg P) \cdot [v(c/P) \oplus v(\neg R)] \leq \frac{1}{2}$.

or

- $B = \{<P, c, 0>\}$,
- $R$ is a relevant undercutter for $B$, i.e. $w(\neg c/P\wedge\neg\neg R) > w(\neg c/P)$,
- $v_{B,R}(c) = v(\neg P) \cdot [v(\neg c/P) \oplus v(\neg R)] \leq \frac{1}{2}$.

Note: an unsuccessful undercutting attack can result in strengthening of the attacked argument if its attacker happens to be non-acceptable and the corresponding hybrid argument is stronger than the attacked one.