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DIMENSION CALCULATION UNCERTAINTIES IN QUEUING MODELS

WU Ning, Ph.D.

Institute for Traffic Engineering, Ruhr-University Bochum, 44780 Bochum, Germany; phone: +49 234 3226557, fax: +49 234 3214151; e-mail: ning.wu@rub.de

Abstract

In order to conduct traffic quality assessments, queuing models are used for estimating delays and queue lengths. By applying the formulas in existing highway capacity manuals a dimension inconsistence of parameters is determined. This inconsistence of dimension calculations leads to incorrect calculation results if traffic flow volumes are converted from the unit veh/h into the unit pcu/h. Such unit conversion can directly affect the calculation results of delays and queue lengths. Thus, it can also affect the estimated traffic qualities. This problem is addressed and clarified with some examples. It turns out that the queuing models used in traffic engineering are basically correct. But for applications of those models certain pre-conditions have to be carefully taken into account.

Keywords: queuing model, dimension calculation, personal car equivalent

Introduction

Queuing models are used for calculating delays and queue lengths. First of all, the standard queuing models of types M/M/1 and M/D/1 and their subtypes are applied in traffic engineering. Also the generalized queuing model from Pollaczek-Khintchine (cf. Stark and Nicholls, 1972) is often used. The deviation of arrivals and service times for a real traffic system can be then considered sufficiently. However, the standard queuing models are only valid for the stationary assumption. Therefore, time-dependent (instationary) queuing models, which also take into account traffic conditions with partly/temporal over-saturations are developed. The most well-known time-dependent queuing models are those from Kimber-Hollis (1979) and Akcelik (1980) which are also incorporated in the highway capacity manuals HBS 2001 (FGSV, 2001) and HCM 2000 (TRB, 2000).

By using the mentioned highway capacity manuals some discrepancies in the parameter dimensions are found in the queuing models. This leads to an effect that the conversion between different units for traffic volumes (veh or pcu) directly influences the results of queue and delay calculations. Such problems have raised uncertainties for applications of those queuing models and doubts about the correctness of the delay and queue length formulas in the existing highway capacity manuals. In this paper, those problems are addressed in details.

Explanation of the problems

At first, a stochastic, stationary queuing system is considered. A stationary queuing system means that the average traffic volume *q* and the capacity *C* are constant over the time and the traffic volume *q* is always lower than the capacity *C*. For example, for an M/M/1 queuing system, the average queue length *N* (cf. Stark and Nicholls, 1972) is expressed as

$$N = \frac{x^2}{1-x} \quad , \tag{1}$$

where *x* is the degree of saturation of the queuing system. In the convention of traffic engineering, the parameter *x* is expressed as the quotient of the traffic volume *q* to the capacity *C*. That is, $x = q / C$.

Since, according to the convention in the traffic engineering, the traffic volume *q* and the capacity *C* have always the same dimension (unit, e.g. veh/s or veh/h), the degree of saturation *x* must be dimensionless. That is, in a mathematical formulation:

$$x[\text{-}] = \frac{q[\text{veh/s}]}{C[\text{veh/s}]} \tag{2}$$

Thus, the average queue length

$$N[\text{-}] = \frac{x^2[\text{-}]}{1[\text{-}] - x[\text{-}]} \tag{3}$$

is also dimensionless. According to the Rule of Little the average delay is

$$w[\text{s/veh}] = \frac{N[\text{-}]}{q[\text{veh/s}]} \tag{3a}$$

Here, the average delay has the dimension [s/veh]. In order to take into account the stochastic properties of a real traffic system, an additional constant $0 \le C_0 \le 1$ is often applied to eq. (3) for pragmatic reasons (cf. Kimber - Hollis, 1979 and Akcelik 1980). Normally, the value of C_0 is also treated as dimensionless. Thus, the average queue length and the average delay under stationary conditions are

$$N[\text{-}] = \frac{C_0[\text{-}]x^2[\text{-}]}{1[\text{-}] - x[\text{-}]} \quad \text{and} \quad w[\text{s/veh}] = \frac{C_0[\text{-}]N[\text{-}]}{q[\text{veh/s}]} \quad . \tag{4}$$

Now, a deterministic queuing system with temporal over-saturation (i.e. time-dependent or instationary) is considered. The average queue length *N* within a time period *T* under consideration can be obtained by establishing an input-output analysis. For example, the average queue length can be expressed by (cf. Kimber - Hollis, 1979 and Akcelik 1980)

$$N[\text{veh}] = \frac{1}{2}\left(q[\text{veh/s}] - C[\text{veh/s}]\right)\cdot T[\text{s}] = \frac{1}{2}C[\text{veh/s}]\cdot\left(\frac{q[\text{veh/s}]}{C[\text{veh/s}]} - 1[\text{-}]\right)\cdot T[\text{s}] \tag{5}$$

The average queue length here has the dimension [veh]. According to Akcelik (1980), the average delay under time-dependent conditions is

$$w[\mathrm{s}] = \frac{N[\mathrm{veh/s}] \cdot T[\mathrm{s}]}{C[\mathrm{veh/s}] \cdot T[\mathrm{s}]} = \frac{N[\mathrm{veh}]}{C[\mathrm{veh/s}]} \tag{5a}$$

The average delay here has the dimension [s]. In order to take into account the queued vehicles after the time period T under consideration, the average delay w is calculated here differently to the Rule of Little (cf. eq. (3)).

There are apparently two different dimensions for the average queue length ([-] and [veh]) and for the average delay ([s/veh and [s]) regarding two different queuing systems. Which dimension do the average queue length and the average delay really have? In the literature, the two queuing systems mentioned above (eq. (3) or (4) and eq. (5)) are combined with each other through the so-called Transit Technique in order to derive time-dependent delay formulas. That unavoidably leads to apparent "inconsistent" dimension calculations in the resulting queue and delay formulas.

In HCM 2000 (TRB, 2000), a delay formula based on the Transit Technique is incorporated (TRB, 2000; eq.17-38) for unsignalized intersections. The dimension calculation for this delay formula is:

$$w = \frac{3600}{C[\mathrm{veh/h}]} + 900T[\mathrm{h}]\left[\frac{q[\mathrm{veh/h}]}{C[\mathrm{veh/h}]} - 1 + \sqrt{\left(\frac{q[\mathrm{veh/h}]}{C[\mathrm{veh/h}]} - 1\right)^2 + \frac{\left(\frac{3600}{C[\mathrm{veh/h}]}\right)\frac{q[\mathrm{veh/h}]}{C[\mathrm{veh/h}]}}{450T[\mathrm{h}]}}\right] \qquad [\text{inconsistent}] \tag{6}$$

The dimension calculation in this formula is inconsistent. The problem in the dimension calculations also leads to different results of queue lengths and delays if a unit conversion is conducted.

In HBS 2001 (FGSV, 2001) the delay formula from Kimber-Hollis (1979) is incorporated. The dimension calculation for this delay formula (FGSV, 2001, page 7-81; cf. also Kimber - Hollis, 1979) is also inconsistent.

Normally, the traffic volume q and the capacity C can be converted from the unit veh/s to the unit pcu/s by a general factor f. That is,

$$q_{pcu} = q_{veh} \cdot f \quad \text{and} \quad C_{pcu} = C_{veh} \cdot f \quad \text{with } f \geq 1 \tag{7}$$

Thus,

$$x_{veh} = \frac{q_{veh}}{C_{veh}} = \frac{q_{veh} \cdot f}{C_{veh} \cdot f} = \frac{q_{pcu}}{C_{pcu}} = x_{pcu} \tag{8}$$

In general, by applications of queue length and delay formulas with different units (pcu and veh) for the traffic volume the following relationships have to be satisfied:

$$N_{pcu} = N_{veh} \cdot f \quad \text{and} \quad w_{pcu} = \frac{N_{pcu}}{q_{pcu}} = \frac{N_{veh} \cdot f}{q_{veh} \cdot f} = w_{veh} \tag{9}$$

Unfortunately, for the stochastic, stationary queuing system (cf. eq. (3) or eq.(4)), we have

$$N_{pcu} = N_{veh} \text{ and } w_{pcu} = \frac{N_{pcu}}{q_{pcu}} = \frac{N_{veh}}{q_{veh} \cdot f} = \frac{w_{veh}}{f} \tag{10}$$

That is also implausible. In average the delay could not be shorter if the traffic volume is converted from veh/s to pcu/s. A required queue space is also not shorter if the calculation of queue length is carried out in pcu instead in veh (including heavy vehicles). The conditions from eq. (9) are not fulfilled.

For the deterministic, time-dependent queuing system (cf. eq.(5)), the conditions from eq. (9) are satisfied.

The time-dependent delay formulas in the existing capacity manuals are established by combining eq. (3) or eq.(4) with eq. (5). Thus, the conversion from the unit veh to the unit pcu leads also there to an incorrect delay and queue length estimation. In general, the conversion from veh to pcu leads to an under-estimation of average delay and queue length. This under-estimation corresponds approximately to the reciprocal of the conversion factor *f*. Such considerable under-estimations cannot be neglected. The influences of those delay under-estimations on the LOS can be under-estimated by up to one whole LOS.

Finding the reasons of the problems

Looking into the derivation of the generalised standard delay formula from Pollazcek-Khinshine one can determine that for calculating the average number of arrivals during the service time the expression $E(m) = q \cdot t_B = x$ is used (cf. Start and Nicholls, 1972, page 426). Where t_B is the average service time and q is the traffic volume. In the further process of the derivation always $x = E(m)$ is used. Therefore, the parameter x is equivalent to the parameter $E(m)$. Thus, the parameter $x_{st} = E(m)$ $= q[\text{veh/s}] \cdot t_B\,[\text{s}]$ for the standard queuing system has here the dimension [veh] not the dimension [-]. The capacity in a standard queuing system is expressed as $C_{st} = 1/t_B$. That is, the corresponding "capacity" for the standard queuing system $C_{st} = 1/t_B$ has the dimension [1/s] not [veh/s].

From this definition, the queue length in the queuing system according to the Pollazcek-Khinshine formula is then

$$N[\text{veh}] = \frac{x_{st}{}^2[\text{veh}^2] + q^2[\text{veh}^2/\text{s}^2]\sigma_{t_B}{}^2[\text{s}^2]}{2(1[\text{veh}] - x_{st}[\text{veh}])} \tag{11}$$

with $\sigma_{t_B}{}^2[\text{s}^2]$ = variance of the service time t_B. The queue length here has also the dimension [veh].

For an M/M/1 queuing system the service time is exponentially distributed. The mean value of the service times is equal to the standard deviation. That is, $\sigma_{t_B}[\text{s}] = t_B[\text{s}]$ and therefore

$$N[\text{veh}] = \frac{x_{st}{}^2[\text{veh}^2] + x_{st}{}^2[\text{veh}^2]}{2(1[\text{veh}] - x_{st}[\text{veh}])} = \frac{x_{st}{}^2[\text{veh}^2]}{1[\text{veh}] - x_{st}[\text{veh}]} = \frac{x^2[\text{-}]}{1[\text{-}] - x[\text{-}]} \cdot 1[\text{veh}] \qquad (12)$$

In the equation above, the parameter x_{st}[veh] can be transferred to x[-] by multiplying a constant 1 with the dimension [veh] to x. The dimension for x_{st} is passed to the constant 1.

For the eq. (3) the expressions with corresponding dimensions for delays and queue lengths are

$$N_{pcu} = \frac{x^2[\text{veh}^2 \cdot f^2]}{1[\text{veh} \cdot f] - x[\text{veh} \cdot f]} = \frac{x^2[\text{veh}^2] \cdot f[\text{-}]}{1[\text{veh}] - x[\text{veh}]} = N_{veh} \cdot f \qquad (13)$$

$$w_{pcu} = \frac{x^2[\text{veh}^2 \cdot f^2]}{q[\text{veh} \cdot f](1[\text{veh} \cdot f] - x[\text{veh} \cdot f])} = \frac{x^2[\text{veh}^2]}{q[\text{veh}](1[\text{veh}] - x[\text{veh}])} = w_{veh} \qquad (14)$$

Now the correct relationship between the converted units is restored.

Writing out the eq. (4) with the corresponding dimensions yields

$$N[\text{-}] = \frac{x^2[\text{veh}^2] \cdot C_0[\text{-}]}{1[\text{veh}] - x[\text{veh}]} = \frac{x^2[\text{-}]}{1[\text{-}] - x[\text{-}]} \cdot C_0[\text{veh}] \qquad (15)$$

That is, the parameter C_0 must have the dimension [veh] if the parameter x is considered dimensionless and the dimension calculation remains consistent. The dimension of x is passed to the constant C_0.

It has to be emphasized, that for a stochastic, stationary standard queuing system (e.g. eq. (3) and eq. (4)) we have always $x_{st}[\text{veh}] = t_B[\text{s}] \cdot q[\text{veh/s}] = q[\text{veh/s}] / C_{st}[1/\text{s}]$ instead of $x[\text{-}] = q[\text{veh}/s] / C[\text{veh/s}]$. That is, $x_{st}[\text{veh}] = x[\text{-}] \cdot 1[\text{veh}]$. If the parameter $x = x_{st}$ is considered as dimensionless, a constant with the dimension [veh] must be added to. For a deterministic, instationary queuing system the derivation of delay formulas (see eq. (5), cf. also Kimber-Hollis, 1979 and Akcelik 1980) uses $x_{inst}[\text{-}] = q[\text{veh/s}] / C[\text{veh/s}] = x[\text{-}]$.

If the dimensions of both differently defined x (x_{st} and x_{inst}) are taken along for deriving the time-dependent delay and queue length formulas, the dimension calculation is then correct in the resulted expressions. For example, the delay formula with all corresponding dimensions for unsignalized intersections in HCM 2000 (TRB, 2000; eq.17-38) is now

$$w = \frac{3600}{C_{st}[1/\text{h}]} + 900T[\text{h}] \left[\frac{q[\text{veh/h}]}{C[\text{veh/h}]} - 1 + \sqrt{\left(\frac{q[\text{veh/h}]}{C[\text{veh/h}]} - 1\right)^2 + \frac{C_0[\text{veh}]\left(\frac{3600}{C[\text{veh/h}]}\right)\frac{q[\text{veh/h}]}{C[\text{veh/h}]}}{450T[\text{h}]}} \right] [\text{s}] \qquad (16)$$

Normally, C_0[veh]=1[veh] is applied. If the traffic volume q and the capacity C are converted from the unit veh/h into pcu/h, $C_0[\text{pcu}] = q_{\text{pcu}}/q_{\text{veh}} \cdot 1[\text{pcu}] = f \cdot 1[\text{pcu}]$ applies. It should be pointed out that the average delay w always has the dimension [s] not the dimension [s/veh] or [s/pcu] and the queue length N always has the dimension [veh] or [pcu] not the dimension [-].

Summary and Conclusions

It is evident that by applications of queue length and delay formulas with different units (pcu and veh) for the traffic volume the following relationships have to be satisfied:

$$N_{pcu} = N_{veh} \cdot f \quad \text{and} \quad w_{pcu} = \frac{N_{pcu}}{q_{pcu}} = \frac{N_{veh} \cdot f}{q_{veh} \cdot f} = w_{veh}$$

All delay and queue length formulas in the existing highway capacity manuals (e.g. HBS 2001; HCM 2000) do not satisfy those relationships. The dimension calculations in such formulas are not consistent. The reason is that the dimensions of the input parameters were wrongly interpreted or neglected by the derivation. A clarification regarding the dimensions in all those queue length and delay formulas is required. In general, converting the input parameters (traffic volume q and capacity C) from the unit veh to pcu leads to under–estimations of average queue lengths and delays. The traffic quality can be in some cases under-estimated by one LOS.

The mentioned uncertainties in dimension calculations and in unit conversions from veh to pcu can be avoided if the queue length and delay formulas in the highway capacity manuals are applied using the unit veh as it is already the case in HCM 2000. If the capacity is only given in the unit pcu/h, e.g. sometimes by calculating capacities at unsignalized intersections, the influence of heavy vehicles can be taken into account by converting the capacity into the unit veh/h. If the results are further used in the unit pcu (e.g. for calculating the length of queuing space), they can then be re-converted from the unit veh to the unit pcu.

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