Estimation of Queue Lengths and Their Percentiles at Signalized Intersections

by Ning Wu

Abstract

Queue lengths are important parameters in traffic engineering for determining the capacity and traffic quality of traffic control equipment. At signalized intersections, queue lengths at the end of red time (red-end) are of greatest importance for dimensioning the lengths of lane. While the average queue length reflects the capacity of traffic signals, the so-called 95th and 99th percentile of queue lengths at red-ends are used for determining the length of turning lanes, such that the risk of a blockage in the through lanes could be minimized. Furthermore, lengths of back-of-queue (queue length at queue-end) must be considered for determining the lengths of turning lanes at signalized intersections.

The queue lengths and their distribution can be numerically calculated from Markov chains. The percentiles of queue lengths can be estimated from the distribution. Based on the results of Markov chains, regressions are undertaken for obtaining explicit formulas under stationary traffic conditions. For non-stationary traffic conditions, the formulas can be derived using the so-called transition techniques.

Key-words: Traffic signals, Queue length, Percentiles of queue lengths, Stationary and non-stationary traffic, Free and bunched traffic

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1 INTRODUCTION

Queue lengths are important parameters in traffic engineering for judging the capacity and traffic quality of traffic control equipment. At signalized intersections, queue lengths at the end of red time (red-end, or RE) are very important for dimensioning the length of lanes. While the average queue length reflects the capacity of traffic signals, the so-called 95th and 99th percentile of queue lengths at RE are usually used for determining the lengths of turning lanes, such that a blockage in the through lanes could be avoided as far as possible. By the 95th or 99th percentile of queue lengths, one means the length of queue that should not be exceeded in 95% or 99% of all cycles respectively. In other words, only in 5% or 1% of all cycles the queue length is larger. At traffic signals, queue lengths at queue-ends (lengths of back-of-queue, or QE) must also be considered. Queue lengths at green-ends (GE) are not critical, but they are basic parameters for calculating the queue lengths at RE and QE. Generally, the average queue length at RE or GE can be determined from queuing theory. The average queue length and the average delay under stationary traffic can be converted from each other by the rule of Little: *queue length = delay • traffic flow*. Under non-stationary traffic a certain relationship between the average queue length and the average delay also exists (Akcelik 1980). For the calculation of average queue lengths at GE and RE, there exist theories from several authors for different traffic conditions (Webster 1958; Miller 1968; Kimber and Hollis 1979; Akcelik 1980; Wu 1990).

The estimate of the 95th and 99th percentile of queue lengths at RE (or QE) is much more difficult. Until now no suitable analytical solutions have been obtained. Webster (1958) compiled with help of simulations two tables for estimating the 95th and 99th percentile of queue lengths at RE under stationary traffic conditions. Pöschl and Waglechner (1982) repeated the simulation with a more capable computer and slightly modified Webster's tables. The tables of Webster and Pöschl-Waglechner are up to now the only sources for estimating the 95th and 99th percentile of queue lengths at RE under stationary traffic condition. With these tables, repeated interpolations and/or extrapolations must be used. This is very impractical for computer calculations and for a manual calculation it is unwieldy and susceptible to errors. For non-stationary traffic, Akcelik and Chung (1994) obtained also by simulations a set of equations for calculating the percentiles (90th, 95th, and 98th percentile) of queue lengths at RE and QE (back-of-queue). These equations tend to over-estimate the percentiles of queue lengths, especially for saturation degree $x > 0.8$. The reason of this over-estimation may be in the assumption that the percentiles of queue lengths can always be expressed as a manifold of the average queue length over the entire range of the saturation degree x. This is not always plausible, especially for $x > 0.8$ under non-stationary traffic conditions.

In this paper, a series of theoretical-empirical functions that represent the 95th and 99th percentile of queue lengths at RE (or QE) under stationary and non-stationary traffic is presented. Bunching in the traffic flow is also considered. The queue length for stationary traffic can be determined by regressions. The data base of the regression was calculated from Markov chains (Wu 1990). The distribution function of the queue lengths can be then determined for each selected point within the cycle time. The numerically determined values are exact under the model conditions. For the 95th and

99th percentile of queue lengths at RE (or QE) under non-stationary traffic conditions, the functions are determined from the so-called transition technique (Kimber and Hollis 1979). The bunching of the traffic flow is considered with a correction factor subjected to the queue length (Wu 1990).

The following symbols are used:

95% queue length			
	=	95th percentile of queue lengths	
	=	queue length, which is not exceeded in 95% of the cycles	(veh)
99% queue length			
	=	95th percentile of queue lengths	
	=	queue length, which is not exceeded in 99% of the cycles	(veh)
W	=	average delay per vehicle	(s/veh)
N_{GE}	=	average queue length at green-end (GE)	(veh)
N_{GE95}	=	95% queue length at GE	(veh)
N_{GE99}	=	99% queue length at GE	(veh)
N_{RE}	=	average queue length at red-end (RE)	(veh)
N_{RE95}	=	95% queue length at RE	(veh)
N_{RE99}	=	99% queue length at RE	(veh)
N_{QE}	=	average queue length at queue-end (back-of-queue, or QE)	(veh)
N_{QE95}	=	95% queue length at QE	(veh)
N_{QE99}	=	99% queue length at QE	(veh)
G	=	length of green time	(s)
R	=	length of red time	(s)
R'	=	apparent red time at QE	(s)
C	=	length of cycle time = $R + G$	(s)
q	=	traffic flow	(veh/s)
λ	=	green time ratio = G/C	(-)
n	=	number of lanes	(-)

s	=	saturation traffic flow	(veh/s)
x	=	saturation degree	(-)
$\bar{x}$	=	average saturation degree during the peak period under non-stationary traffic	(-)
c	=	capacity per cycle = $s \bullet G$	(veh)
Q	=	capacity of the traffic signal = c/C	(veh/s)
T	=	length of the peak period	(s)
l	=	vehicle spacing at rest	(m)
xx	=	index for average, 95%, or 99% queue length	
XX	=	index for green-end (GE), red-end (RE), queue-end (QE)	
in	=	index for non-stationarity (or instationarity)	
Kg	=	correction factor for queue length under bunched traffic	(-)
m	=	Factor for randomness of the traffic flow (normally m=1)	(-)
Vs'	=	Speed of vehicles leaving a queue	(m/s)
Vq'	=	Speed of vehicles pulling into a queue	(m/s)

2 QUEUE LENGTHS AT RED-END UNDER STATIONARY AND FREE TRAFFIC

Generally, the queue lengths $N_{REx x}$ (average, 95%, and 99% queue length) at RE can be expressed as functions of queue length N_{GExx} at GE, the traffic flow q, the cycle time C, and the red time R. Therefore, a theoretical-empirical formula of the form

$$N_{RExx} = N_{GExx}(\alpha) + \beta \cdot q \cdot R + \gamma \cdot (q \cdot C)^n \tag{1}$$

where the parameters α, β, γ, and n are different for the average, 95%, and 99% queue lengths respectively, can be used as the regression function for queue lengths at RE. Under stationary and free traffic, the queue lengths N_{GExx} (average, 95%, and 99% queue lengths) at GE can be estimated as the product of the average queue length N_{GE} at GE. That is,

$$N_{GExx}(\alpha) = \alpha \cdot N_{GE} \tag{2}$$

Eq. (1) consists of 3 terms. The first term describes the queue length at GE. It is a function of the average queue length N_{GE} at GE. The second term is the increase of the queue length between GE and RE. This term depends on the number of vehicles that arrive during the red time R. The third term is a correction factor accounting for the randomness of the traffic flow. It depends on the number of vehicles arriving during a cycle time C. The first and the third term represent the stochastic part of queue lengths; the second term describes the deterministic part. The parameters α, β, γ, and n can be determined through regressions in order to fit the database obtained from Markov chains.

For the average, 95%, and 99% queue lengths, 560 combinations of green time G (G = 10-50 s with increments of 10 s), saturation degree x (x = 0.3-0.98 with increments of 0.02), and cycle time C (C = 60-90 s with increments of 10 s) were calculated according to the theory of Markov chains (Wu 1990). The regression parameters α, β, γ, and n were determined by the method of the smallest error squares. The obtained parameters for eq. (1) are shown in Tab. 1.

Queue length	α	β	γ	n	standard deviation s
N_{RE}	1.00	1.00	0.00	0.00	0.027
N_{RE95}	2.97	1.20	1.29	0.26	0.291
N_{RE99}	4.65	1.19	1.84	0.39	0.601

Tab. 1 - Parameters for eq. (1)

Tab. 1 shows, that the deviations between values from regression and values from calculation according to Markov chains are always below 1 vehicle (standard deviation $s < 1$).

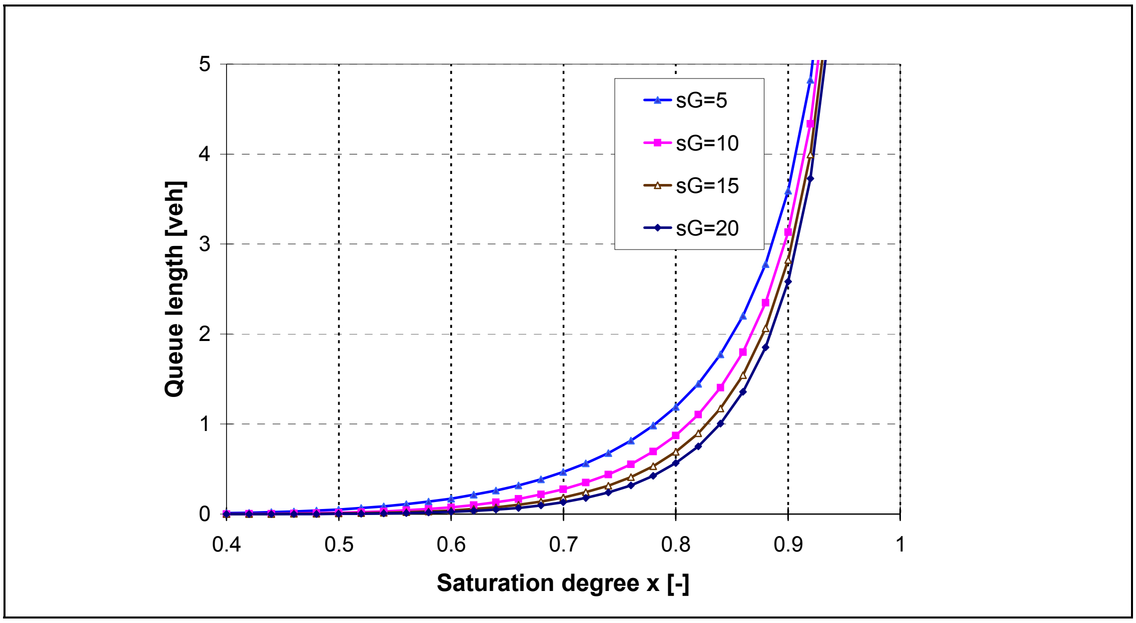

Fig. 1 - Queue lengths at GE under stationary a and free traffic conditions

The average queue length N_{GE} at GE can be calculated from Miller (1968):

$$N_{GE} = \frac{\exp(-1.33\sqrt{s \cdot G} \cdot (1-x)/x)}{2(1-x)} \tag{3}$$

Eq. (3) was verified by the results from Markov chains. An very good agreement between eq. (3) and Markov chains was found (Brilon and Wu 1990; Wu 1990). In Fig. 1, the shape of the queue length N_{GE} at GE as a function of the saturation degree x is illustrated. It shows that the queue length N_{GE} at GE increases with decreasing values of $s \cdot G$.

Inserting eq. (3) and the parameters in Tab. 1 into eq. (1), one obtains

$$N_{RE} = \frac{\exp(-1.33\sqrt{s \cdot G} \cdot (1-x)/x)}{2(1-x)} + q \cdot R \tag{4}$$

$$N_{RE95} = \frac{2.97 \cdot \exp(-1.33\sqrt{s \cdot G} \cdot (1-x)/x)}{2(1-x)} + 1.20 \cdot q \cdot R + 1.29(q \cdot C)^{0.26} \tag{5}$$

and

$$N_{RE99} = \frac{4.65 \cdot \exp(-1.33\sqrt{s \cdot G} \cdot (1-x)/x)}{2(1-x)} + 1.19 \cdot q \cdot R + 1.84 \cdot (q \cdot C)^{0.39} \tag{6}$$

The relationship between N_{RE}, N_{RE95}, and N_{RE99} is shown in Fig. 2. In this example, $C = 60$ s, $G = 20$ s, $s = 0.5$ veh/s, and $q = 0.1$ veh/s are used.

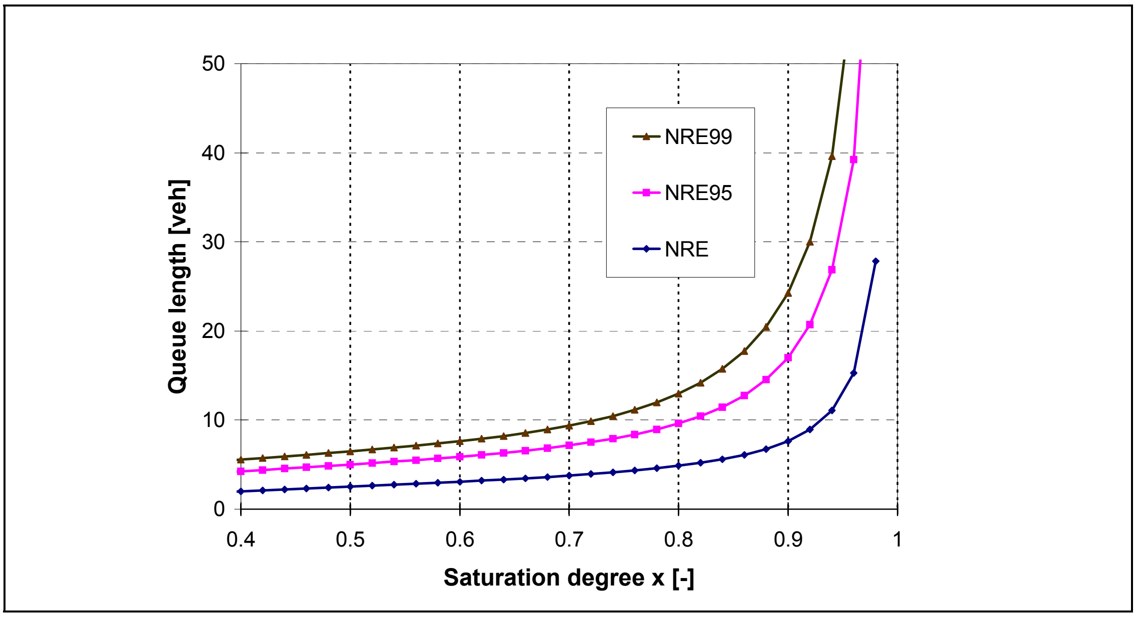

Fig. 2 - Queue lengths at RE under stationary and free traffic conditions

	x	c / λ	2		5		10		15		20		30		40	
			S	R	S	R	S	R	S	R	S	R	S	R	S	R
95% queue lengths at RE	0.30	0.2	2	2	3	3	5	5	7	7	9	8	12	11	15	14
		0.4	2	2	3	3	4	4	6	6	7	7	9	9	12	12
		0.6	1	2	2	3	3	4	4	5	5	5	7	7	9	9
		0.8	1	2	1	2	2	3	3	3	3	4	4	5	5	6
	0.50	0.2	3	3	5	5	8	7	10	10	13	12	18	18	23	23
		0.4	3	3	4	4	6	6	8	8	10	10	14	14	18	18
		0.6	2	3	3	3	5	5	6	6	8	8	10	10	13	13
		0.8	2	2	2	3	3	4	4	4	5	5	6	7	8	8
	0.70	0.2	5	5	7	7	10	10	14	14	17	17	24	24	31	31
		0.4	5	5	6	6	9	8	11	11	14	14	19	19	24	24
		0.6	4	5	5	5	7	7	8	8	10	10	14	14	17	17
		0.8	4	4	4	5	4	5	5	6	6	7	8	9	10	10
	0.80	0.2	8	8	9	10	13	13	17	17	20	20	28	28	35	35
		0.4	7	8	8	9	11	11	14	14	16	16	22	22	27	28
		0.6	7	7	7	8	9	9	11	11	12	13	16	16	20	20
		0.8	6	7	6	7	7	7	7	8	8	9	10	10	12	12
	0.90	0.2	15	16	17	17	20	21	24	24	28	28	35	36	43	44
		0.4	15	15	16	16	18	19	21	21	24	24	29	30	35	36
		0.6	14	15	15	15	16	16	18	18	20	20	23	23	27	27
		0.8	14	14	14	14	14	14	15	15	16	15	17	17	19	18
	0.95	0.2	30	31	32	32	35	36	39	39	43	43	50	51	58	59
		0.4	29	30	31	31	33	33	36	36	39	39	44	44	50	50
		0.6	29	30	30	30	31	31	33	33	35	34	38	38	42	41
		0.8	29	29	29	29	29	29	30	29	31	30	32	31	34	32
99% queue lengths at RE	0.30	0.2	3	3	4	4	7	6	9	8	11	10	14	13	17	17
		0.4	2	2	4	4	6	5	7	7	9	8	11	11	14	14
		0.6	2	2	3	3	4	5	6	6	7	7	9	9	11	11
		0.8	2	2	2	3	3	4	4	5	4	6	6	7	7	8
	0.50	0.2	4	4	6	6	9	9	12	12	15	15	21	20	26	25
		0.4	4	4	5	5	8	8	10	10	12	12	17	17	21	21
		0.6	3	4	4	5	6	6	8	8	9	10	12	13	15	16
		0.8	3	3	3	4	4	5	5	6	6	7	8	9	9	11
	0.70	0.2	7	7	9	9	13	12	16	16	20	20	27	27	34	34
		0.4	7	7	8	8	11	11	14	13	16	16	22	22	27	27
		0.6	7	7	7	7	9	9	11	11	12	13	16	17	20	21
		0.8	6	6	6	6	7	7	7	8	8	10	10	12	12	14
	0.80	0.2	11	11	13	13	17	16	20	20	24	24	31	32	39	39
		0.4	11	11	12	12	15	14	17	17	20	20	26	26	31	32
		0.6	10	11	11	11	13	13	14	14	16	16	19	20	23	24
		0.8	10	10	10	10	11	11	11	11	12	12	13	14	15	17
	0.90	0.2	23	23	25	25	28	28	32	32	35	35	43	43	50	51
		0.4	22	23	24	24	26	26	29	28	31	31	37	37	42	43
		0.6	22	23	23	23	24	24	26	25	27	27	31	30	34	34
		0.8	22	22	22	22	22	22	23	22	23	22	25	24	26	26
	0.95	0.2	45	47	48	48	51	51	55	55	58	58	66	66	73	74
		0.4	45	46	47	47	49	49	52	51	54	54	60	59	65	65
		0.6	44	46	46	46	47	47	49	48	50	49	54	53	57	56
		0.8	44	45	45	45	45	44	46	45	46	45	48	46	49	47

S - Simulation from Pöschl   R - Regression from eq. (5) and (6)

41 - deviation = 1 vehicle   23 - deviation = 2 vehicle

Tab. 2 - Queue length at RE

Comparing the results of eqs. (5) and (6) with the tables from Pöschl and Waglechner (1982), where values are obtained from simulations, one can see, that the agreement is amazingly good (Tab. 2). The absolute deviation totals maximally 2 vehicles. The deviation is evidently the result of rounding of numbers and of stochastic-conditional deviations of the simulation model.

3 QUEUE LENGTHS AT QUEUE-END (QE) UNDER STATIONARY AND FREE TRAFFIC

Eqs. (5) and (6) describe the 95% and 99% queue lengths at RE under stationary and free traffic conditions. However, the queue length at RE is not the maximum back-of-queue length within the cycle. While the queue length discharges forwards from the stop line after the beginning of green time, the queue length still increases backwards at end of the queue. More vehicles still have to stop after the RE (green-begin). For a vehicle approaching to the traffic signal, the red time appears longer than it is. Denoting the time from GE (red-begin) up to the instant that the queue length is discharged completely as R', and inserting R' into eqs. (5) and (6), one obtains the 95% and 99% queue length at QE (back-of-queue).

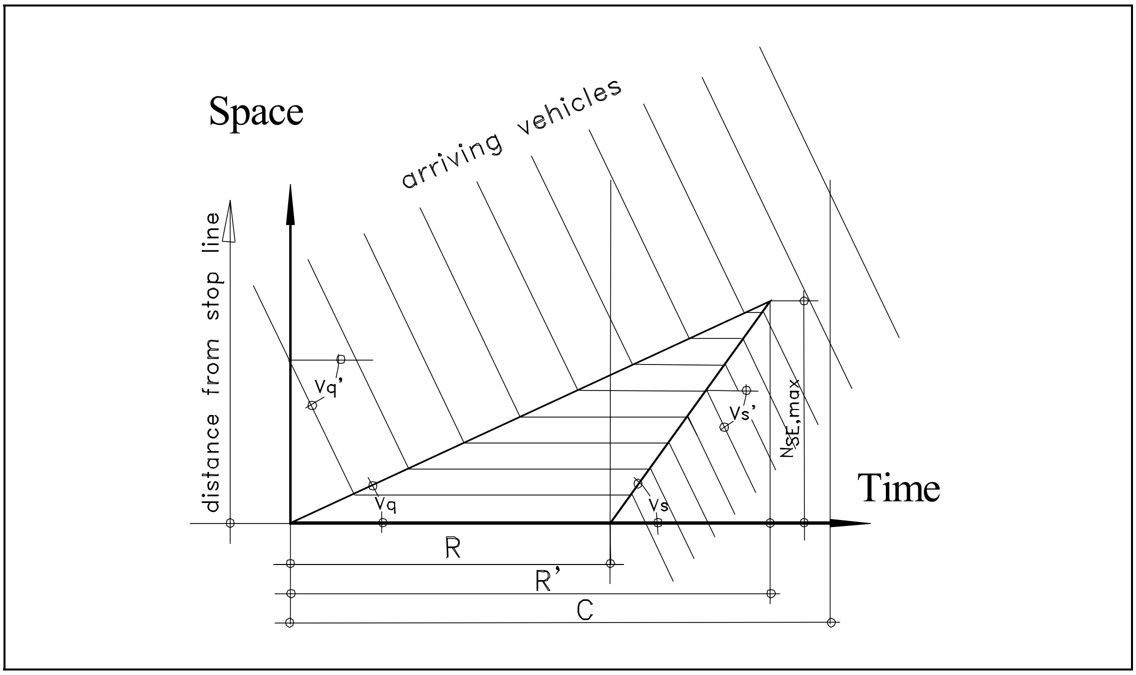

Fig. 3 - Queue length at queue-end (in this figure is $N_{SE,max}$=N_{QE})

The value of R' can be determined from continuum theory. Here, only the so-called deterministic queue length is considered. The deterministic queue length from the stop line up to the queue-end (QE) can be represented by a time-space-diagram (Fig. 3). Here, Vs is the speed that the queue is discharged away from the stop line and Vq the speed that the queue at the queue-end increases backwards. Assuming that vehicles leave the front of the queue with speed Vs' and pull in behind of the queue with speed Vq', one obtains according to the continuum theory

$$Vq = \frac{0-q}{\frac{1}{l} - \frac{q}{Vq'}} \qquad\qquad Vs = \frac{s-0}{\frac{s}{Vs'} - \frac{1}{l}} \tag{7}$$

From the geometry of the time-space-diagram (Fig. 3), R' can be written as

$$R' = \frac{R}{1 - \frac{q}{s} \cdot \left(\frac{1}{l} - \frac{s}{Vs'}\right) \Big/ \left(\frac{1}{l} - \frac{q}{Vq'}\right)} \tag{8}$$

For Vq' and Vs', the average speed the vehicles move away from the queue and into the queue (in meters per second) should be used. Rewriting eq. (8) in the following form

$$R' = \frac{1 - \frac{q}{s}}{1 - \frac{q}{s} \cdot \left(\frac{1}{l} - \frac{s}{Vs'}\right) \Big/ \left(\frac{1}{l} - \frac{q}{Vq'}\right)} \cdot \frac{R}{1 - \frac{q}{s}}$$

and setting

$$K = \frac{1 - \frac{q}{s}}{1 - \frac{q}{s} \cdot \left(\frac{1}{l} - \frac{s}{Vs'}\right) \Big/ \left(\frac{1}{l} - \frac{q}{Vq'}\right)} \tag{9}$$

one obtains

$$R' = K \cdot \frac{R}{1 - \frac{q}{s}} \tag{10}$$

The parameter K depends on the speeds Vq' and Vs' and on the traffic flows q and s. Inserting $Vs' = Vq' = 11.11$ m/s (40 km/h) and $l = 6$ m into eq. (9), one obtains K between 1 (with $q = 0$) and 0.73 (with $q = s$). The shape of K as a function of the traffic flow q is presented in Fig. 4. Note that K is a linear function of q for the case $Vs' = Vq'$.

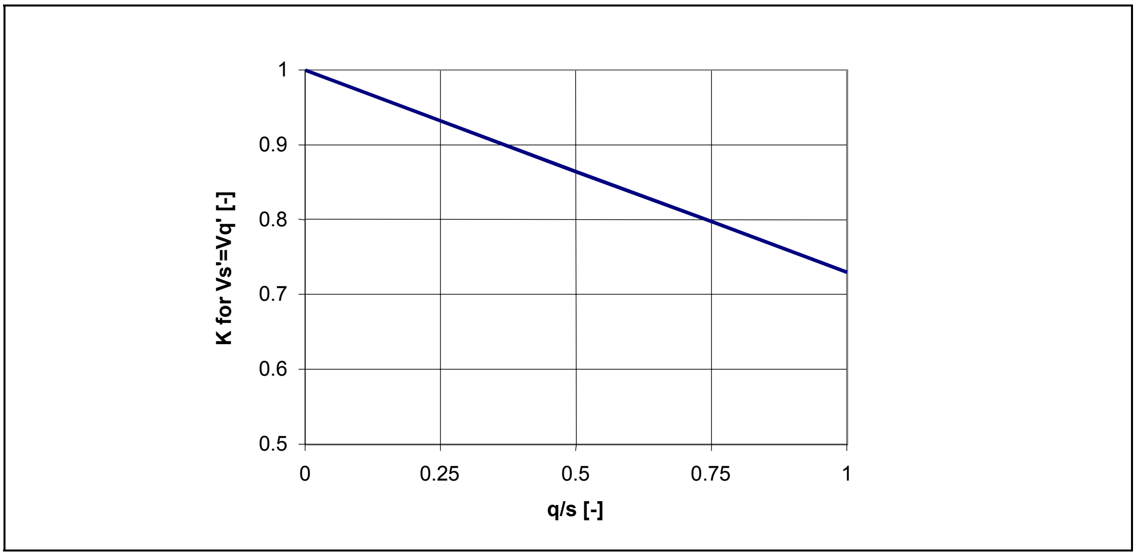

Fig. 4 - Factor K for queue lengths at QE (back-of-queue)

In the practice, the traffic flow q lies usually in the area between 1/4- and 1/2-fold of the saturation traffic flow s. In order to simplify eq. (10), K can be replaced by its average value in this area. The average value in this area is approximately 0.9. Eq. (10) can be then rewritten as

$$R' = 0.9 \cdot \frac{R}{1 - \frac{q}{s}} \tag{11}$$

Also, Akcelik (1980) proposed a factor 0.9 for accounting for the back-of-queue. However, his finding was based on practical observations. Replacing R within eq. (1) with R' (eq. (10)), one obtains a general formula for queue lengths at QE under stationary and free traffic conditions:

$$\begin{aligned} N_{QExx} &= N_{GExx}(\alpha) + \beta \cdot q \cdot K \cdot \frac{R}{1 - \frac{q}{s}} + \gamma \cdot (q \cdot C)^n \\ &= \alpha \cdot N_{GE} + \beta \cdot q \cdot K \cdot \frac{R}{1 - \frac{q}{s}} + \gamma \cdot (q \cdot C)^n \end{aligned} \tag{12}$$

Inserting the α, β, γ, and n values into eq. (12), the 95% and 99% queue length at QE yield

$$N_{QE95} = 2.97 \cdot N_{GE} + 1.20 \cdot q \cdot K \cdot \frac{R}{1 - \frac{q}{s}} + 1.29 \cdot (q \cdot C)^{0.26} \tag{13}$$

and

$$N_{QE99} = 4.65 \cdot N_{GE} + 1.19 \cdot q \cdot K \cdot \frac{R}{1-\frac{q}{s}} + 1.84 \cdot (q \cdot C)^{0.39} \tag{14}$$

The average queue length at QE is (cf. also Akcelik 1980)

$$N_{QE} = N_{GE} + q \cdot K \cdot \frac{R}{1-\frac{q}{s}} \approx N_{GE} + 0.9 \cdot q \cdot \frac{R}{1-\frac{q}{s}} \tag{15}$$

4 QUEUE LENGTHS UNDER NON-STATIONARY TRAFFIC CONDITIONS

Under non-stationary traffic conditions, the traffic flow does not remain constant over time. Queue lengths of all types (average, 95%, and 99% queue lengths) are accordingly also dependent on time. In traffic engineering, one is interested only in a certain period of time, namely the peak period (or rush-hour). The pattern of the traffic flow in this period is predefined (e.g., as a rectangle or a parabola). The average values of queue lengths and delays should be determined for this period.

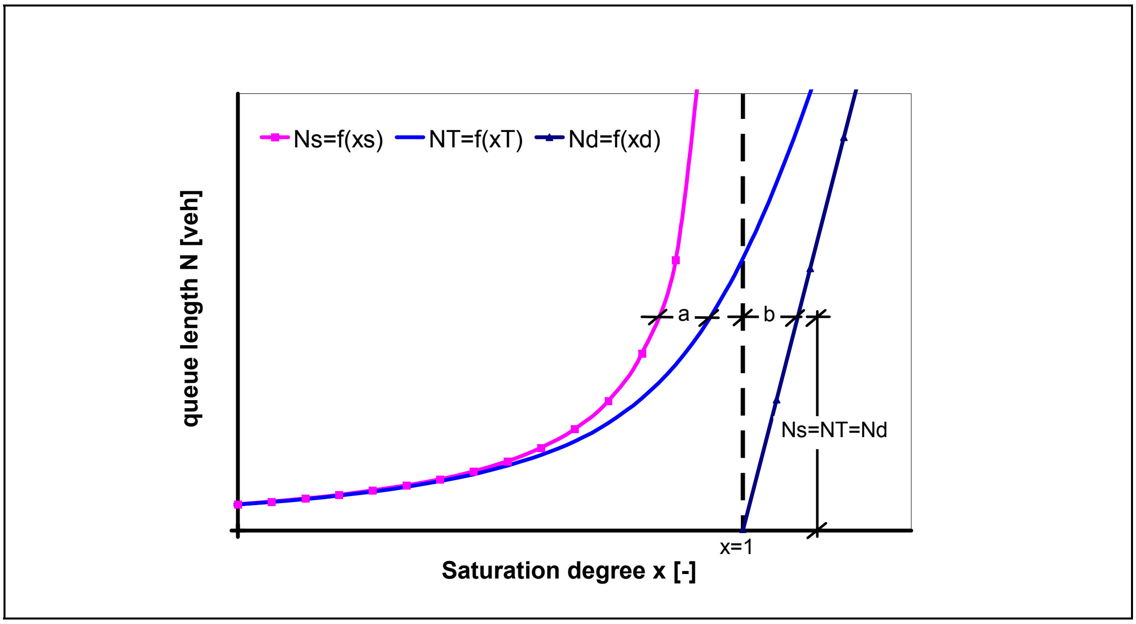

Fig. 5 - Principle of the transition technique

The queue lengths under non-stationary traffic conditions can be determined by the so-called transition technique introduced by Kimber and Hollis (1979). This technique can be illustrated with Fig. 5.

In Fig. 5, the stochastic queue length N_s for stationary traffic, the deterministic queue length N_d at overloaded state ($x > 1$) for non-stationary traffic, and the total queue length N_T (stochastic + deterministic) for non-stationary and stochastic traffic are shown together. N_T is the parameter to be found by the transition technique. The principle of the transition technique is the postulate $a = b$ for the equal queue length

$$N_s = N_T = N_d = n \tag{16}$$

More realistically, one should postulate that instead the absolute distance $a = b$ the relative distance $a/x_{s,max=1} = b/x_d$. That is, one postulates

$$\frac{\text{distance } a}{1} = \frac{\text{distance } b}{x_d} \qquad \Rightarrow \qquad \frac{1-x_s}{1} = \frac{x_d - x_T}{x_d} \tag{17}$$

Eq. (17) yields the relationship

$$x_T = x_s \cdot x_d \tag{18}$$

Using for N_s different formulae of queue lengths in stationary traffic (average, 95%, and 99% queue lengths), one obtains accordingly for N_T the transited approximate formulae for calculating queue lengths under non-stationary traffic conditions. The deterministic queue length at the overloaded state N_d is a constant function. It is the same for every percentile. The parameter N_d depends on the length of the peak period T and has the value

$$N_d = \begin{cases} (x_d - 1) \cdot \dfrac{Q \cdot T}{2} & \text{for } x_d \geq 1 \\ 0 & \text{for } x_d < 1 \end{cases} \tag{19}$$

To facilitate the transition, a simplification for calculating the average queue length N_{GE} at GE in stationary traffic is meaningful and necessary. Here, instead of eq. (3), the approximation

$$N_{s,GE} = \frac{m \cdot x_s}{(1 - x_s)} \cdot \frac{2}{\sqrt{s \cdot G}} \tag{20}$$

is used. In eq.(20), m is a parameter for the randomness that depends on the coordination of traffic signals and controller settings. It should be validated by measurements. Normally, one can use $m = 0.5$.

Solving eq. (19) for x_d and eq. (20) for x_s and setting $x_d = f(N_d)$ and $x_s = f(N_s)$ into eq. (18), one obtains

$$\left(N_d \cdot \frac{2}{Q \cdot T} + 1 \right) \cdot \frac{N_{s,GE}}{\dfrac{2 \cdot m}{\sqrt{s \cdot G}} + N_{s,GE}} = x_T \tag{21}$$

Setting $N_d = N_{s,GE} = N_{GE,in}$ and $x_T = \bar{x}$ and solving for $N_{GE,in}$ yields

$$N_{GE,in} = \frac{Q \cdot T}{4} \cdot \left(\bar{x} - 1 + \sqrt{(\bar{x}-1)^2 + \frac{8 \cdot m \cdot \bar{x}}{Q \cdot T} \cdot \frac{2}{\sqrt{s \cdot G}}} \right) \tag{22}$$

This equation should be used for the rang of $4 \le s \cdot G \le 40$. Certainly, also other formulae (Akcelik 1980; Kimber and Hollis 1979; Brilon and Wu 1990; Wu 1990; HCM 2000 (Catalina Engineering, Inc. 1997)) for calculating the queue length $N_{GE,in}$ at GE under non-stationary traffic conditions can be used. For instance, the formula of Wu (1990) which is based on a parabola traffic pattern for the peak period is used in the German Highway Capacity Manual (Brilon, Grossmann and Blanke 1994).

Eq. (22) can also be used for calculating the average delay for non-stationary traffic conditions. The average delay in the peak period can be calculated with the formula (Akcelik 1980)

$$W = \frac{C \cdot (1-\lambda)^2}{2(1-\lambda \cdot \bar{x})} + \frac{N_{GE,in}}{Q} \tag{23}$$

Replacing eq. (20) with $N_{s,GExx}(\alpha) = \alpha \bullet N_{s,GE}$ for the 95% and 99% queue lengths, one obtains the corresponding formulae for calculating the 95% and 99% queue lengths at GE for non-stationary and free traffic conditions:

$$N_{GExx,in}(\alpha) = \frac{Q \cdot T}{4} \cdot \left(\bar{x} - 1 + \sqrt{(\bar{x}-1)^2 + \alpha \cdot \frac{8 \cdot m \cdot \bar{x}}{Q \cdot T} \cdot \frac{2}{\sqrt{s \cdot G}}} \right) \tag{24}$$

The only difference in eq. (24) compared with eq. (22) is that the second term within the square is multiplied by the parameter α. In agreement with the delay formula in the proposed 2000 HCM (Catalina Engineering, Inc. 1997), using

$$N_{s,GE} = \frac{m \cdot x_s}{(1-x_s)}$$

instead eq.(20) one obtains the equation

$$N_{GExx,in}(\alpha) = \frac{Q \cdot T}{4} \cdot \left(\bar{x} - 1 + \sqrt{(\bar{x}-1)^2 + \alpha \cdot \frac{8 \cdot m \cdot \bar{x}}{Q \cdot T}} \right) \tag{25}$$

for calculating the queue length $N_{GExx,in}$ at GE.

According to the proposed 2000 HCM (Catalina Engineering, Inc. 1997) one can use $m = k \cdot l$ to take the effect of coordination and controller settings into account. The values of k and l are given in the proposed 2000 HCM (Catalina Engineering, Inc. 1997).

Using eqs. (1) and (12), the general formulae for the queue lengths at RE and QE for non-stationary and free traffic can be written as:

$$N_{REx x,in} = N_{GExx.in}(\alpha) + \beta \cdot q \cdot R + \gamma \cdot (q \cdot C)^n \tag{26}$$

and

$$N_{QExx,in} = N_{GExx.in}(\alpha) + \beta \cdot q \cdot K \cdot \frac{R}{1-\frac{q}{s}} + \gamma \cdot (q \cdot C)^n \tag{27}$$

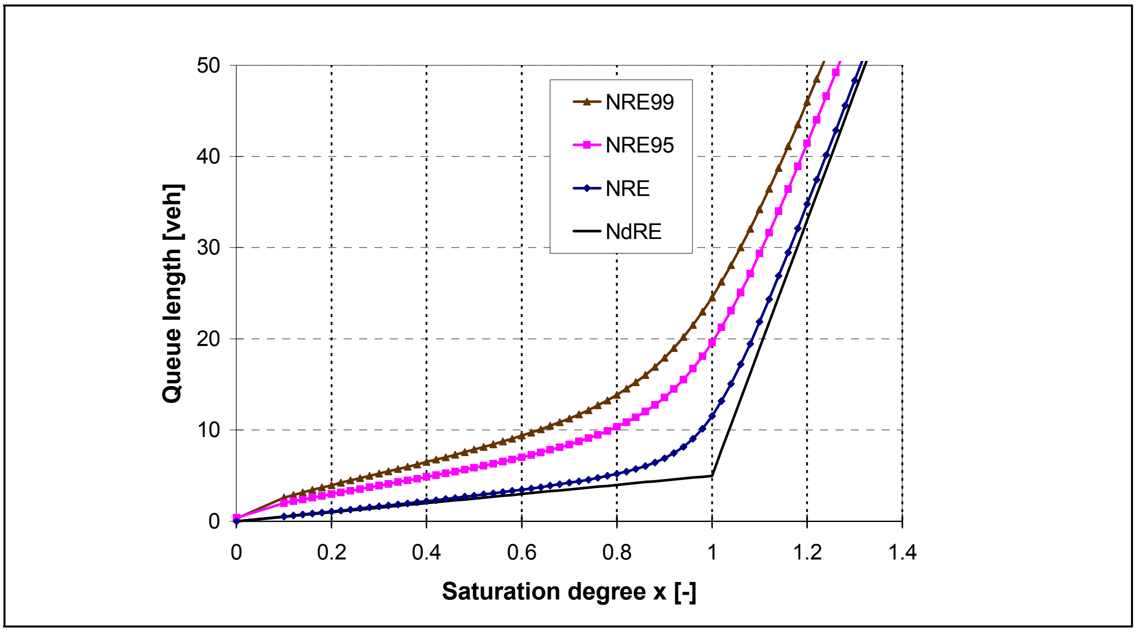

Fig. 6 - Queue lengths at RE under non-stationary and free traffic conditions

In Fig. 6, the deterministic (N_{dRE}), the average (N_{RE}), the 95% (N_{RE95}), and the 99% (N_{RE99}) queue lengths at RE for non-stationary and free traffic are illustrated.

5 ACCOUNTING FOR BUNCHING EFFECTS WITHIN THE ARRIVAL PATTERNS

The assumption that the moving vehicles have no influence on each other - described by the concept "free traffic" - is not always correct for real world traffic conditions, especially on busy single-lane streets. A vehicle in heavy traffic must watch out for the vehicle ahead and leave a time and space headway to it. In this case, the traffic is bunched. The smallest possible time headway for traffic approaching signalized intersections is denoted as the minimal arrival headway τ. The bunching of traffic flow can be estimated by a factor Kg for calculating queue lengths at traffic signals (Wu 1990):

$$Kg = 1 - \frac{1-(1-q\cdot\tau)^2}{2-x} \tag{28}$$

where τ is the minimal arrival headway between consecutive vehicles. Depending on the type of vehicles and driving behavior, the value of τ could be different. If the variance of the distribution of the minimal arrival headway τ is known, the factor Kg can be calculated as follows (Wu 1990):

$$Kg = 1 - \frac{1-(1-\bar{\tau}\cdot q)^2 - q^2\cdot\sigma_\tau^2}{2-x} \tag{29}$$

According to Böhm (1968), an average minimal arrival headway $\bar{\tau} = 1.6$ s was observed. The variance σ_τ^2 of the Erlang-distributed minimal arrival headway τ is accordingly:

$$\sigma_\tau^2 = \frac{\bar{\tau}^2}{6} = \frac{1.6^2}{6} = 0.43\ s^2$$

Inserting $\bar{\tau} = 1.6$ s and $\sigma_\tau^2 = 0.43$ s^2 into eq. (29), one obtains

$$\begin{aligned} Kg &= 1 - \frac{1-(1-1.6\cdot q)^2 - q^2\cdot 0.43}{2-x} \\ &= 1 - \frac{3.2\cdot q - 3\cdot q^2}{2-x} \end{aligned} \tag{30}$$

Thus Kg is always between 0 and 1, i.e., the queue length is reduced by bunching of traffic flow. Replacing the factor α by $\alpha' = \alpha\cdot Kg$ in the equations, one obtains the formula for calculating the queue length accounting for bunching effects in the arriving traffic flow.

6 OTHER PERCENTILES OF QUEUE LENGTHS

In the earlier sections, equations were introduced for calculating the 95% and 99% queue lengths for different traffic conditions. Other percentiles of queue lengths can be determined using these parameters through an appropriate interpolation / extrapolation function.

Theoretically, the distribution function of queue lengths should be used as the interpolation / extrapolation function. Since this distribution function of queue lengths at signalized intersections is unknown, an approximate function is introduced for representing the distribution of queue lengths. Normally, one can assume that the distribution function $F(n)$ of queue lengths n can be approximately expressed through the function (Wu 1994):

$$F(n) = 1 - x^{A(B\cdot n+1)} \tag{31}$$

where A and B are parameters to be determined. This approximate function yields reasonable results for calculating the distribution of queue lengths, especially in the area where the subjected queue length n is larger than the average queue length N.

Inserting the 95% and 99% queue lengths into eq. (31), one obtains

$$\begin{cases} F(N_{95}) = 0.95 = 1 - x^{A(B \cdot N_{95}+1)} \\ F(N_{99}) = 0.99 = 1 - x^{A(B \cdot N_{99}+1)} \end{cases} \tag{32}$$

which correspond to

$$\begin{cases} 0.05 = x^{A(B \cdot N_{95}+1)} \\ 0.01 = x^{A(B \cdot N_{99}+1)} \end{cases} \tag{33}$$

Solving the system (33) for A and B one obtains

$$A = \frac{\ln(0.05) \cdot N_{99} - \ln(0.01) \cdot N_{95}}{\ln(x) \cdot (N_{99} - N_{95})}$$

$$B = \frac{\ln(0.01) - \ln(0.05)}{\ln(0.05) \cdot N_{99} - \ln(0.01) \cdot N_{95}}$$

Inserting A and B into eq. (31), one then obtains the φ-th percentile of queue lengths

$$\begin{aligned} N_{\varphi} &= N_{|F(n)=\varphi/100} \\ &= \frac{1}{A \cdot B}\left(\frac{\ln(1-F(n))}{\ln(x)} - A\right) \\ &= N_{95} - \left(1.86 + \frac{\ln(1-\frac{\varphi}{100})}{1.61}\right) \cdot (N_{99} - N_{95}) \end{aligned} \tag{34}$$

For example, one can determine the 90% and 98% queue lengths using eq. (34):

$$\begin{aligned} N_{85} &= N_{95} - \left(1.86 + \frac{\ln(1-0.85)}{1.61}\right) \cdot (N_{99} - N_{95}) \\ &= N_{95} - 0.68 \cdot (N_{99} - N_{95}) \end{aligned}$$

$$\begin{aligned} N_{98} &= N_{95} - \left(1.86 + \frac{\ln(1-0.98)}{1.61}\right) \cdot (N_{99} - N_{95}) \\ &= N_{95} - 0.57 \cdot (N_{99} - N_{95}) \end{aligned}$$

7 SUMMARY

queue length		α'	β'	γ'	n'
average value	N_{GE}	$1.00 \cdot Kg$	0	0	0
	N_{RE}		1.00		
	N_{QE}		$K/(1-q/s)$		
95%-value	N_{GE95}	$2.97 \cdot Kg$	0	1.29	0.26
	N_{RE95}		1.20		
	N_{QE95}		$1.20 \cdot K/(1-q/s)$		
99%-value	N_{GE99}	$4.65 \cdot Kg$	0	1.84	0.39
	N_{RE99}		1.19		
	N_{QE99}		$1.19 \cdot K/(1-q/s)$		

Tab. 3 - Parameters for queue lengths at signalized intersections

Equations for calculating queue lengths at signalized intersections under different traffic conditions are derived. The general formula for calculating the queue length can be expressed as

$$N_{XXxx} = N_{GExx}(\alpha') + \beta' \cdot q \cdot R + \gamma' \cdot (q \cdot C)^{n'} \tag{35}$$

For different traffic conditions, the parameters α', β', γ', and n' are assembled in Tab. 3. $N_{GExx}(\alpha')$ is in general a function of α'. For stationary traffic, $N_{GExx}(\alpha')$ is

$$N_{GExx}(\alpha') = \alpha' \cdot \frac{\exp(-1.33\sqrt{s \cdot G} \cdot (1-x)/x)}{2(1-x)} \tag{36}$$

For non-stationary traffic, $N_{GExx}(\alpha')$ is

$$N_{GExx}(\alpha') = \frac{Q \cdot T}{4} \cdot \left(\bar{x} - 1 + \sqrt{(\bar{x}-1)^2 + \alpha' \cdot \frac{8 \cdot m \cdot \bar{x}}{Q \cdot T} \cdot \frac{2}{\sqrt{s \cdot G}}} \right) \tag{37}$$

or in agreement with the delay formula in the proposed 2000 HCM (Catalina Engineering, Inc. 1997)

$$N_{GExx}(\alpha') = \frac{Q \cdot T}{4} \cdot \left(\bar{x} - 1 + \sqrt{(\bar{x} - 1)^2 + \alpha' \cdot \frac{8 \cdot m \cdot \bar{x}}{Q \cdot T}} \right) \tag{38}$$

Other parameters in Tab. 3 are

$$K = \frac{1 - \frac{q}{s}}{1 - \frac{q}{s} \cdot (\frac{1}{l} - \frac{s}{Vs'}) \Big/ (\frac{1}{l} - \frac{q}{Vq'})} \approx 0.9 \tag{39}$$

and

$$Kg = \begin{cases} 1 - \dfrac{3.2 \cdot q - 3 \cdot q^2}{2 - x} & \text{for single lane traffic} \\ 1 & \text{for multilane traffic} \end{cases} \tag{40}$$

These formulae should be used according to Tab. 4.

	traffic in peak period	traffic in normal period
multi-lane street	non-stationary and free traffic eq. (37) or (38) and $Kg = 1$	stationary and free traffic eq. (36) and $Kg = 1$
Single lane street	non-stationary and bunched traffic eq. (37) or (38) and $Kg \neq 1$	stationary and bunched traffic eq. (36) and $Kg \neq 1$

Tab. 4 - Recommendations for the application of the formula

Here, the "peak period" means that the traffic flow before and after the considered time interval is significantly, i.e., at least 15%, smaller than the traffic flow within the considered time interval. For multi-lane streets, it is assumed that the traffic is distributed at the stop line evenly over all lanes. The "normal period" means that before, within, and after the considered time interval the traffic conditions remain constant.

Other φ-th percentiles (85%, 90%, 98% etc.) of queue lengths of all types can be approximately estimated by the interpolation and/or extrapolation function:

$$N_{\varphi} = \begin{cases} N_{95} - (1.86 + \dfrac{\ln(1-\frac{\varphi}{100})}{1.61}) \cdot (N_{99} - N_{95}) & for\ N_{\varphi} > 0 \\ 0 & for\ N_{\varphi} \le 0 \end{cases} \quad (41)$$

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