

Capacity of Shared-Short Lanes at Unsignalised Intersections

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ABSTRACT

The calculation procedures in recent highway capacity manuals do not exactly treat shared/short lanes at unsignalized intersections in an exact manner. The capacity of individual streams (left turn, through and right turn) are calculated separately. If the streams share a common traffic lane, the capacity of the shared lane is then calculated according to the shared lane procedure from Harders (1968), i.e., the lengths of the short lanes are considered either as infinite or as zero. The exact lengths of the separate short lanes cannot be take into account. Therefore, the capacity computed from conventional methods is overestimated, whereas that from the shared lanes formula (as in Chapter 10 of the 1994 *Highway Capacity Manual*) is underestimated. This paper presents an analytical procedure, based on probability theory, for estimating the capacity of this combination of shared and short lanes. This procedure combines the existing procedures for estimating the capacity of shared and short lanes. It was tested by simulations in the style of the KNOSIMO simulation, and it can be used for arbitrary lane configurations. For simple shared/short lane configurations, explicit equations are derived for estimating the capacity. For complicated shared/short lane configurations, iteration procedures are given. As a special case, the so-called flared minor approaches are treated according to the theory derived.

keywords: capacity, unsignalised intersection, short lanes, shared lanes, flared lanes.

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INTRODUCTION

Unsignalized Intersections (cross-roads and T-junctions), where traffic is regulated by traffic signs, are the most commonly used intersections in traffic management. The right-of-way regulated by traffic signs presupposes that a driver makes the decision to pass through if he is at the first waiting position directly at the stop line or if no other vehicle is waiting in front of him. The calculation procedures developed for this situation, which are also used in numerous manuals (1,2,3), are standard for calculating the capacity of unsignalised intersections. The two best known and simplest procedures are these from Harders (4) and Siegloch (5).

The calculation procedures in recent manuals (1,2,3) assume that traffic streams that must give way have their own traffic lanes at the intersection. The capacity of the individual streams (left turn, through and right turn) are calculated separately. If the streams share a common traffic lane, the capacity of the shared lane is then calculated according to the shared lane procedure from Harders (4).

The procedures for considering the lane distribution at unsignalized intersections are based on the assumption that left-turn and/or right-turn streams have either infinitely long exclusive lanes or no exclusive lanes at all. In reality, however, this is not the case. If an approach with short traffic lanes (Figure 1a) for the left-turn and/or right-turn streams is calculated, the capacity is either overestimated (length of the exclusive lanes as infinite) or underestimated (length of the exclusive lanes as zero).

In this paper, a procedure is derived, with which the length of the turn lanes can be considered exactly for calculating the capacity of the shared lane. The precision of this calculation procedure is tested through simulations.

The following symbols and indices are used:

Symbols:

m	= number of sub - streams	[-]
C	= capacity	[vph]

q	= traffic flow	[vph]
x	= degree of saturation	[-]
n	= length of queue space in number of vehicles	[veh]
P_s	= probability that a point on the street is occupied by traffic	[-]
k	= factor for estimating the capacity of shared lane = $1 / x_{sh,real}$	[-]
$x_{sh,real}$	= real degree of saturation of shared lane = q_{sh} / C_{sh}	[-]
C_{sh}	= capacity of shared lane	[vph]
q_{sh}	= traffic flow of shared lane	[vph]
x_{sh}	= apparent degree of saturation of shared lane	[-]

Indices for systems with arbitrarily many sub-streams:

i	= index for the i -th sub-stream
$i1$	= index for the i -th sub-stream of the level 1
$i2$	= index for the i -th sub-stream of the level 2
j	= index for the j -th step of iterations
sh	= index for shared lane
$sh1$	= index for shared lane of the level 1
$sh2$	= index for shared lane of the level 2

Indices for systems with three sub-streams:

L	= index for left-turn streams and their traffic lanes
T	= index for through streams and their traffic lanes
R	= index for right-turn streams and their traffic lanes
LT	= index for shared streams consisting of a left-turn and a through stream and their traffic lanes
TR	= index for shared streams consisting of a through and a right-turn stream and their traffic lanes

MATHEMATICAL DERIVATIONS

In Figure 1a the possible combinations of short traffic lanes are presented. The short traffic lanes at unsignalized intersections have usually two basic forms:

- 1. All three direction streams divide at a point (Figure 1b, type 1)
- 2. The streams divide one after another at two points (Figure 1b, type 2 and 3)

Mathematical derivations are given in this paper for both basic forms of short traffic lanes.

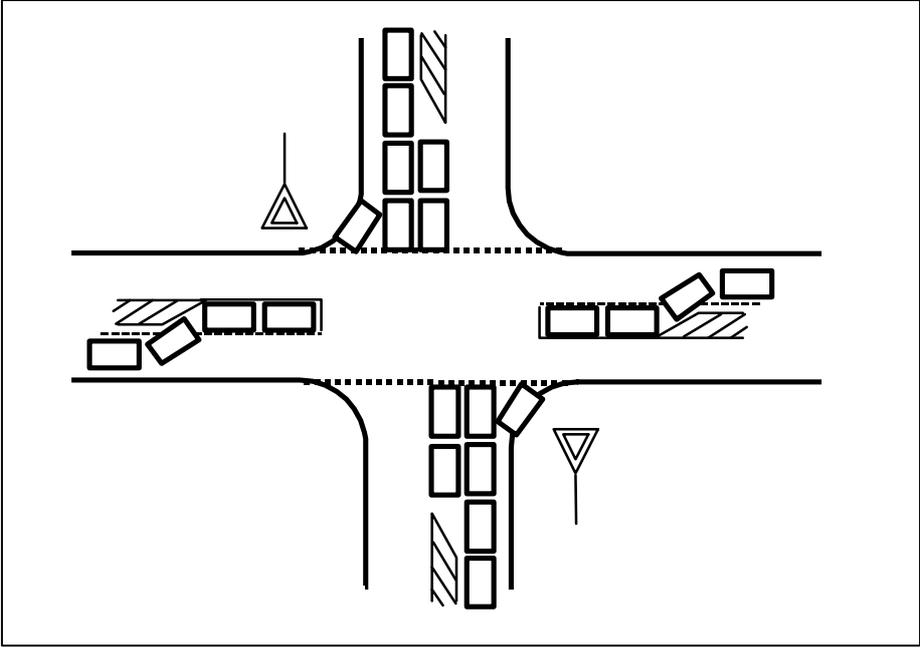


Figure 1a. Possible queues at the approaches of unsignalised intersections

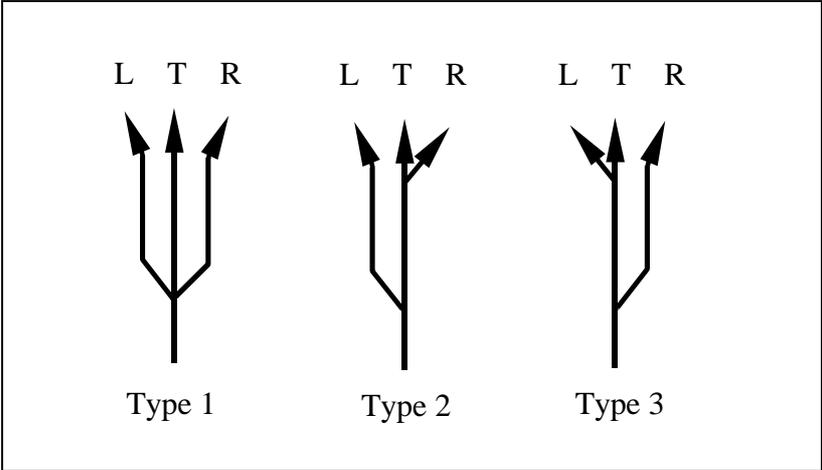


Figure 1b. Combination forms of short traffic lanes

First, a generalized system with m sub-streams, which all develop at the point A from one shared lane (Figure 2), is considered. The sub-stream i is described by the parameters q_i (traffic flow), C_i (capacity) and x_i (degree of saturation). The capacity C_i and the degree of saturation $x_i = q_i / C_i$ are considered under the assumption that there are infinite queue places for the subject stream i . Accordingly, the shared lane has the parameters q_{sh} , C_{sh} and x_{sh} .

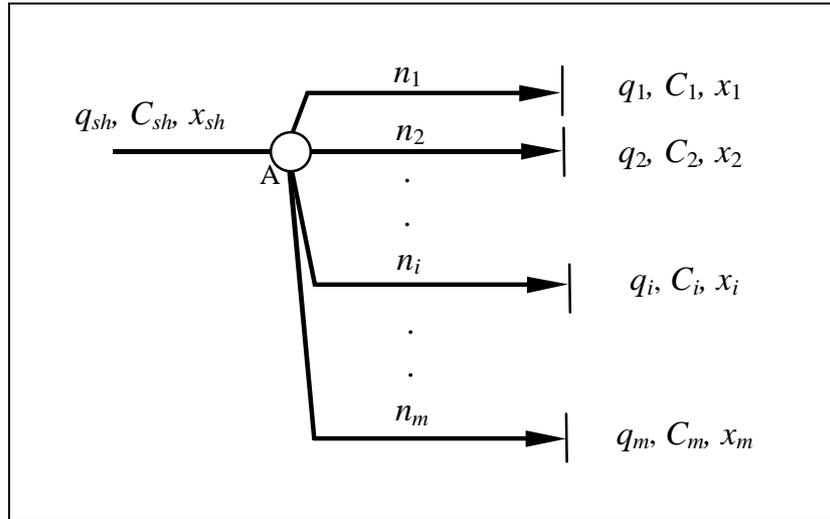


Figure 2. Relationship between a shared lane and its sub - streams

For point A, the following fundamental state condition holds:

Point A is equally occupied from the left (shared lane) and from the right (all sub-streams) by waiting vehicles.

That is, the probability that point A is occupied on the side of the shared lane is equal to the probability that point A is occupied on the side of the sub-streams. It follows that

$$P_{s,sh} = P_{s,1} + P_{s,2} + \dots + P_{s,i} + \dots + P_{s,m} = \sum_{i=1}^m P_{s,i} \quad (1)$$

The probability that point A is occupied by a sub-stream is equal to the probability that the queue length in this sub-stream is larger than the length of the queue space (section from the stop line to point A), i.e., for the sub-stream i ,

$$P_{s,i} = \Pr(N > n_i) \quad (2)$$

The distribution function of queue lengths in a waiting stream at unsignalized intersections can be represented approximately by the following equation (see also Wu (6)):

$$F(n_i) = \Pr(N \leq n_i) = 1 - x_i^{a(bn_i+1)} \quad (3)$$

with

$$x_i = \frac{q_i}{C_i}$$

$a, b =$ parameters

Accordingly one obtains

$$P_{s,i} = \Pr(N > n_i) = 1 - F(n_i) = x_i^{a(bn_i+1)} \quad (4)$$

The M/M/1-queuing system also offers good approximation for the queuing system at unsignalized intersections (see also Wu (6)). In this case one has $a = 1$ and $b = 1$. Thus,

$$P_{s,i} = \Pr(N > n_i) = x_i^{n_i+1} \quad (5)$$

For further derivations, the queuing system at intersections without traffic signals is considered as an M/M/1-queuing system. The resulting deviation can be considered negligible (see also Wu (6)).

If one considers point A as a counter in the sense of queuing system, then the probability that point A is occupied on the side of the shared lane is equal to the degree of saturation of the shared lane, i.e.,

$$P_{s,sh} = \Pr(N > 0) = x_{sh}^{0+1} = x_{sh} \quad (6)$$

Inserting Equations (6) and (5) into Equation (1), one obtains

$$P_{s,sh} = \sum_{i=1}^m x_i^{n_i+1} = x_{sh} \quad (7)$$

Here, however, x_{sh} is only the apparent degree of saturation of the shared lane. That means,

$$x_{sh} \neq \frac{q_{sh}}{C_{sh}}$$

and accordingly one has also

$$C_{sh} \neq \frac{q_{sh}}{x_{sh}} = \frac{\sum_{i=1}^m q_i}{\sum_{i=1}^m x_i^{n_i+1}}$$

The establishment of these inequalities lies in the fact that no linear relationship exists between the traffic flow and the degree of saturation in the shared lane as a result of the exponents of x_i . The capacity of the shared lane can be determined only in other ways.

For estimating the capacity of the shared lane, the following definition is made:

The capacity of the shared lane is the traffic flow at which the merge point A on both sides is occupied 100%

That is, $P_{s,sh,max} = x_{sh,max} = \sum x_i = 1$.

As a rule, the traffic flows q_i (existing or predicted) do not describe the complete saturation of the shared lane. The capacity of the shared lane is generally greater than the sum of q_i (in case of under-saturation by existing q_i). In this case, the traffic flows at the subject traffic stream would approach the limit of the capacity if the q_i values increase. In general, each q_i value could have different increases. It is assumed, however, that an equal increase factor k can be applied for these fictional increases of existing traffic flows. Thus, k is that factor by which all traffic flows on the subject approach has to increase to reach the maximal possible traffic flow: the capacity.

By multiplying the degree of saturation of all sub-streams by this factor k and postulating

$$P_{s,sh,max} = x_{sh,max} = \sum_{i=1}^m (k \cdot x_i)^{n_i+1} \stackrel{!}{=} 1 \quad (8)$$

one obtains the capacity of the subject shared lane

$$C_{sh} = k \cdot q_{sh} = k \cdot \sum_{i=1}^m q_i \quad (9)$$

Accordingly , the real degree of saturation of the shared lane becomes

$$x_{sh,real} = \frac{q_{sh}}{C_{sh}} = \frac{1}{k} \quad (10)$$

Thereby k is determined implicitly by Equation (8). For $n_1 = n_2 = \dots = n_i = \dots = n_m = n$, that is, all sub-streams have the same length of queue space, resulting in

$$k|_{all\ n_i=n} = \frac{1}{\sqrt[n+1]{\sum_{i=1}^m x_i^{n+1}}} \quad (11)$$

and

$$C_{sh}|_{all\ n_i=n} = \frac{\sum_{i=1}^m q_i}{\sqrt[n+1]{\sum_{i=1}^m x_i^{n+1}}} \quad (12)$$

For n_i with general values the Equation (8) cannot be solved explicitly for k . The solution for k can be found, however, according to the Newton method iteratively and numerically. The iteration procedure is

$$k_{j+1} = k_j - \frac{f(k_j)}{f'(k_j)} \quad (j = 0, 1, 2, \dots; k_0=1)$$

with

$$f(k) = \sum_{i=1}^m (k \cdot x_i)^{n_i+1} - 1 \quad (13)$$

The iterations are convergent for all $k > 0$.

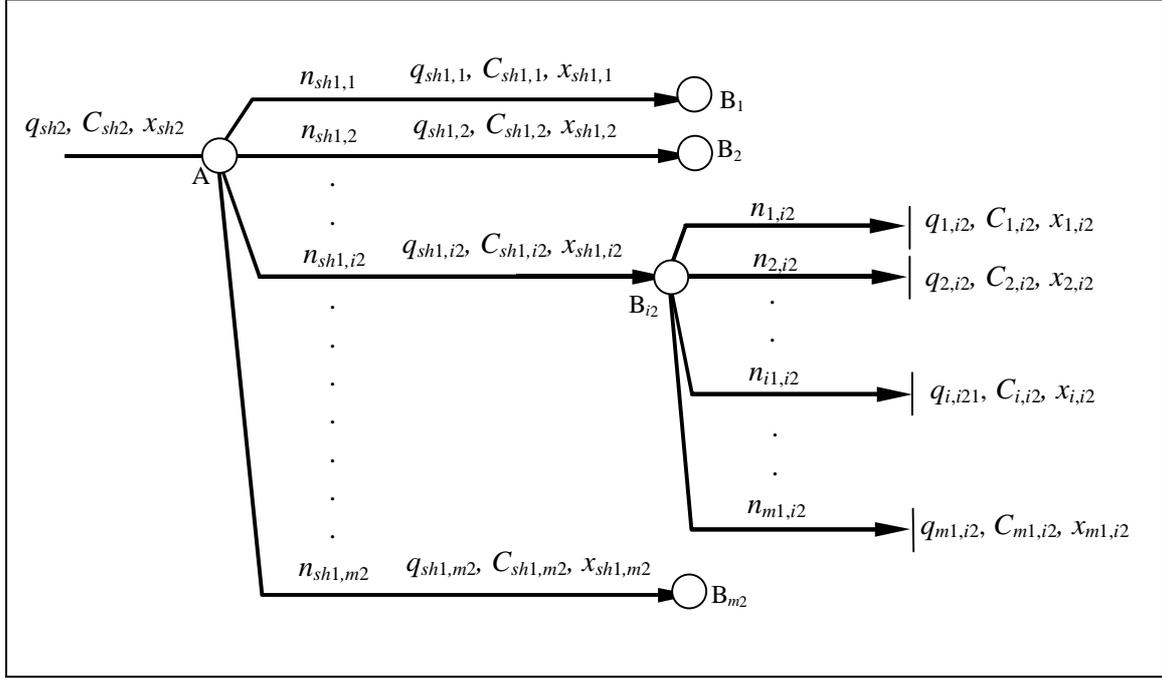


Figure 3. Relationship between shared lanes and their sub- and sub-sub-streams

If a sub-stream again consists of several sub-sub-streams, this sub-stream must be considered as a shared stream itself (Figure 3). Accordingly, analogously to Equation (1), for the merge point A one obtains

$$P_{s,sh2} = \sum_{i2=1}^{m2} P_{s,i2} \quad (14)$$

And for the sub merge points B_{i2} , one obtains

$$P_{s,sh1,i2} = \sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} \quad (15)$$

If one considers the queuing systems in all sub-streams and sub-sub-streams as M/M/1-queuing systems respectively, then for the sub-sub-stream with index $i1, i2$, one obtains

$$P_{s,i1,i2} = x_{i1,i2}^{n_{i1,i2}+1} \quad (16)$$

for the sub merge point with index B_{i2} (section between point A and B_{i2})

$$P_{s,sh1,i2} = x_{sh1,i2} = \sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} = \sum_{i1=1}^{m1_{i2}} x_{i1,i2}^{n_{i1,i2}+1} \quad (17)$$

for the sub-stream with index $i2$

$$P_{s,i2} = x_{sh1,i2}^{n_{sh1,i2}+1} = P_{s,sh1,i2}^{n_{sh1,i2}+1} = \left(\sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} \right)^{n_{sh1,i2}+1} = \left(\sum_{i1=1}^{m1_{i2}} x_{i1,i2}^{n_{i1,i2}+1} \right)^{n_{sh1,i2}+1} \quad (18)$$

and for the merge point A

$$P_{s,sh2} = x_{sh2} = \sum_{i2=1}^{m2} P_{s,i2} = \sum_{i2=1}^{m2} \left(\sum_{i1=1}^{m1_{i2}} P_{s,i1,i2} \right)^{n_{sh1,i2}+1} = \sum_{i2=1}^{m2} \left(\sum_{i1=1}^{m1_{i2}} x_{i1,i2}^{n_{i1,i2}+1} \right)^{n_{sh1,i2}+1} \quad (19)$$

Multiplying the degree of saturation of all sub-sub-streams by a factor k and postulating

$$P_{s,sh2,max} = x_{sh2,max} = \sum_{i2=1}^{m2} \left[\sum_{i1=1}^{m1_{i2}} (k \cdot x_{i1,i2})^{n_{i1,i2}+1} \right]^{n_{sh1,i2}+1} = 1 \quad (20)$$

the capacity of the total shared stream becomes

$$C_{sh} = C_{sh2} = k \cdot q_{sh2} = k \cdot \sum_{i2=1}^{m2} \sum_{i1=1}^{m1_{i2}} q_{i1,i2} \quad (21)$$

The Equations (20), (21), and (22) are the generalized forms of Equations (7), (8), and (9). Setting $m1_{i2}$ or $m2$ equal to 1, one obtains here the Equation (7), (8), and (9) again.

For $n_{i1,i2}$ and $n_{sh1,i2}$ with general values the iteration procedure for solving k becomes

$$k_{j+1} = k_j - \frac{f(k_j)}{f'(k_j)} \quad (j = 0, 1, 2, \dots; k_0=1)$$

with

$$f(k) = \sum_{i2=1}^{m2} \left[\sum_{i1=1}^{m1_{i2}} (k \cdot x_{i1,i2})^{n_{i1,i2}+1} \right]^{n_{sh1,i2}+1} - 1 \quad (22)$$

Analogously, one can also treat systems with an arbitrarily number of levels of sub-sub-streams.

PRACTICAL APPLICATIONS OF THE THEORY

Type 1 lane combinations (see Figure 1b and 4)

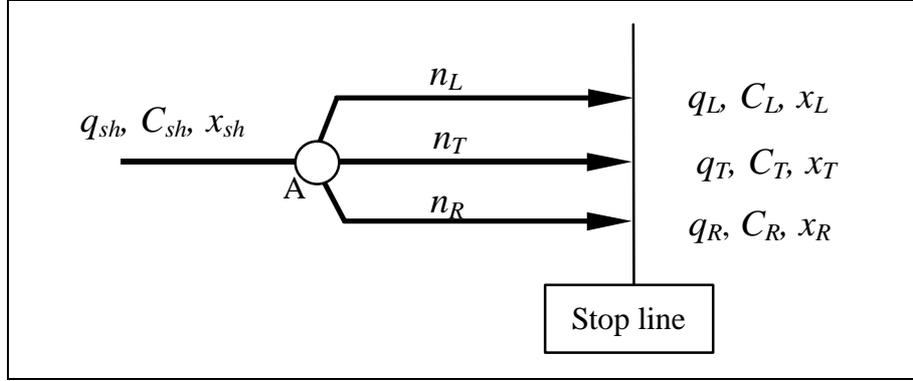


Figure 4. Parameters for Type 1 of short traffic lanes

Setting $m_2 = 1$ and $i_1 = L, T, R$ in Equations (19), (20), and (21), one obtains the following equations for estimating the capacity of the shared lane for Type 1 short lanes at unsignalized intersections:

$$P_{s,sh} |_{type1} = \sum P_{s,i} = \sum P(N > n_i) = \sum x_i^{n_i+1} = x_L^{n_L+1} + x_T^{n_T+1} + x_R^{n_R+1} \quad (23)$$

$$P_{s,sh,max} |_{type1} = x_{sh,max} = \sum (k \cdot x_i)^{n_i+1} = (k \cdot x_L)^{n_L+1} + (k \cdot x_T)^{n_T+1} + (k \cdot x_R)^{n_R+1} = 1 \quad (24)$$

$$C_{sh} |_{type1} = k \cdot q_{sh} = k \cdot (q_L + q_T + q_R) \quad (25)$$

In general, the three streams (L , T , and R) must stop and wait at the same stop line. This means that the numbers of available queue places are equal for all three streams. In this case, setting $n_L = n_T = n_R = n$, one obtains

$$k |_{type1} = \frac{1}{\sum x_i^{n+1}} = \frac{1}{x_L^{n+1} + x_T^{n+1} + x_R^{n+1}} \quad (26)$$

and

$$C_{sh} |_{type1} = k |_{type1} \cdot (q_L + q_T + q_R) = \frac{q_L + q_T + q_R}{x_L^{n+1} + x_T^{n+1} + x_R^{n+1}} \quad (27)$$

At $n = 0$, one gets

$$C_{sh} |_{type1, n=0} = \frac{q_L + q_T + q_R}{x_L + x_T + x_R} \quad (28)$$

That is exactly the well-known shared-lane formula from Harders (4).

For n_L , n_T , and n_R with general values, the iteration procedure for solving k (see also Equation (13)) yields

$$k_{j+1} |_{type1} = k_j - \frac{(k_j \cdot x_L)^{n_L+1} + (k_j \cdot x_T)^{n_T+1} + (k_j \cdot x_R)^{n_R+1} - 1}{(n_L+1) \cdot (k_j \cdot x_L)^{n_L} \cdot x_L + (n_T+1) \cdot (k_j \cdot x_T)^{n_T} \cdot x_T + (n_R+1) \cdot (k_j \cdot x_R)^{n_R} \cdot x_R} \quad (j = 0, 1, 2, \dots; k_0=1) \quad (29)$$

With this equation, the Newton iteration method is to be used for determining the subject k . The capacity of the whole approach can then be obtained according to Equation (9).

Type 2 lane combinations (see Figure 1b and 5)

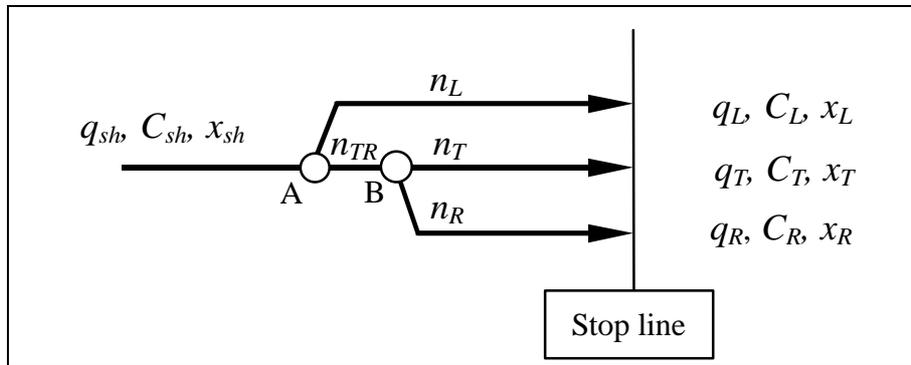


Figure 5. Parameters for Type 2 of short traffic lanes

Setting $i2 = L, TR$ and $i1 = T, R$ in Equations (19), (20), and (21), one obtains the following equation for estimating the capacity of the shared lane for Type 2 short lanes at unsignalized intersections:

$$P_{s,sh} |_{type2} = x_L^{n_L+1} + (x_T^{n_T+1} + x_R^{n_R+1})^{n_{TR}+1} \quad (30)$$

$$P_{s,sh,max} |_{type2} = x_{sh,max} = (k \cdot x_L)^{n_L+1} + [(k \cdot x_T)^{n_T+1} + (k \cdot x_R)^{n_R+1}]^{n_{TR}+1} = 1 \quad (31)$$

$$C_{sh} |_{type2} = k \cdot q_{sh} = k \cdot (q_L + q_T + q_R) \quad (32)$$

Also in this case, the three streams must generally stop and wait at the same stop line. The following relationships exist between the available queue places:

$$n_T = n_R = n_{T,R}$$

$$n_L = n_{TR} + n_{T,R}$$

where $n_{T,R}$ is the common number of queue spaces for left-turn and through streams.

According to these relationships one obtains

$$P_{s,sh} \Big|_{type2, n_T=n_R, n_L=n_{TR}+n_{T,R}} = x_L^{n_L+1} + (x_T^{n_{T,R}+1} + x_R^{n_{T,R}+1})^{n_{TR}+1} \quad (33)$$

$$\begin{aligned} P_{s,sh,max} \Big|_{type2, n_T=n_R, n_L=n_{TR}+n_{T,R}} &= x_{sh,max} \\ &= (k \cdot x_L)^{n_L+1} + \left[(k \cdot x_T)^{n_{T,R}+1} + (k \cdot x_R)^{n_{T,R}+1} \right]^{n_{TR}+1} \\ &= (k \cdot x_L)^{n_L+1} + \left[\left(x_T^{n_{T,R}+1} + x_R^{n_{T,R}+1} \right)^{\frac{1}{n_{T,R}+1}} \cdot k \right]^{n_{TR} \cdot n_{T,R} + n_{TR} + n_{T,R} + 1} \\ &= (k \cdot x_L)^{n_L+1} + \left[\left(x_T^{n_{T,R}+1} + x_R^{n_{T,R}+1} \right)^{\frac{1}{n_{T,R}+1}} \cdot k \right]^{n_{TR} \cdot n_{T,R} + n_L + 1} \\ &\quad \vdots \\ &= 1 \end{aligned} \quad (34)$$

Setting

$$\begin{aligned} x_I &= x_L \\ n_I &= n_L \\ x_{II} &= \left(x_T^{n_{T,R}+1} + x_R^{n_{T,R}+1} \right)^{\frac{1}{n_{T,R}+1}} \\ n_{II} &= n_{TR} \cdot n_{T,R} + n_{TR} + n_{T,R} \\ &= n_{TR} \cdot n_{T,R} + n_L \end{aligned} \quad (35)$$

one obtains for the postulate (Equation (34))

$$P_{s,sh,max} \Big|_{type2, n_T=n_R, n_L=n_{TR}+n_{T,R}} = x_{sh,max} = (k \cdot x_I)^{n_I+1} + (k \cdot x_{II})^{n_{II}+1} \stackrel{!}{=} 1 \quad (36)$$

This means that under the marginal condition that all three sub-streams stop and wait at the same stop line, the shared-lane system with three sub-streams can be simplified in a shared-lane system with only two sub-streams.

Furthermore, the capacity under this condition is

$$C_{sh} \Big|_{type2, n_T=n_R, n_L=n_{TR}+n_{T,R}} = k \cdot (q_L + q_T + q_R) = \frac{(q_L + q_T + q_R)}{x_{sh,real}} \quad (37)$$

For n_L , n_T , and n_R with general values, the iteration procedure for estimating k yields

$$k_{j+1} = k_j - \frac{a}{b + c \cdot d}, \quad (j = 0, 1, 2, \dots; k_0 = 1) \quad (38)$$

with

$$a = (k_j \cdot x_L)^{n_L+1} + [(k_j \cdot x_T)^{n_T+1} + (k_j \cdot x_R)^{n_R+1}]^{n_{TR}+1} - 1$$

$$b = (n_L + 1) \cdot (k_j^{n_L} \cdot x_L)^{n_L} \cdot x_L$$

$$c = (n_{TR} + 1) \cdot [(k_j \cdot x_T)^{n_T+1} + (k_j \cdot x_R)^{n_R+1}]^{n_{TR}}$$

$$d = [(n_T + 1) \cdot (k_j \cdot x_T)^{n_T} \cdot x_T + (n_R + 1) \cdot (k_j \cdot x_R)^{n_R} \cdot x_R]$$

Type 3 lane combinations (see Figure 1b and 6)

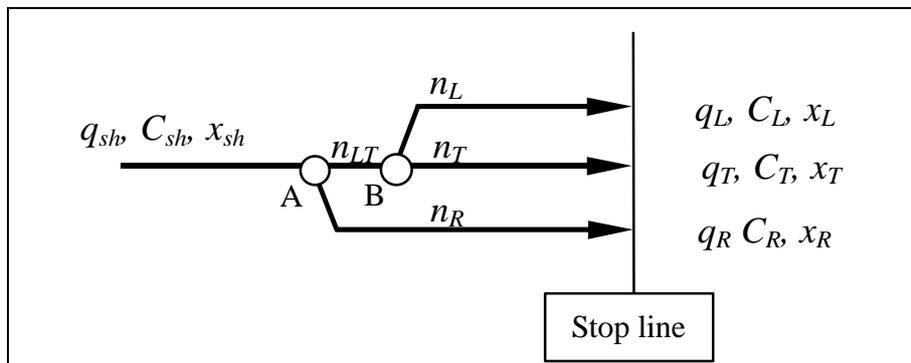


Figure 6. Parameters for Type 3 of short traffic lanes

Type 3 short lanes is symmetrically to Type 2 short lanes. Exchanging index L (left-turn) and R (right-turn) in the equations for Type 2 short lanes yields the corresponding equations for Type 3 short lanes.

For example, analogously to Equations (30), (31), and (32), one obtains here

$$P_{s,sh} |_{type3} = (x_L^{n_L+1} + x_T^{n_T+1})^{n_{LT}+1} + x_R^{n_R+1} \quad (39)$$

$$P_{s,sh,max} |_{type3} = x_{sh,max} = [(k \cdot x_L)^{n_L+1} + (k \cdot x_T)^{n_T+1}]^{n_{LT}+1} + (k \cdot x_R)^{n_R+1} \stackrel{!}{=} 1 \quad (40)$$

$$C_{sh} |_{type3} = k \cdot q_{sh} = k \cdot (q_L + q_T + q_R) \quad (41)$$

Flared lane at minor approaches

Right flared lane at minor approaches (see Figure 7)

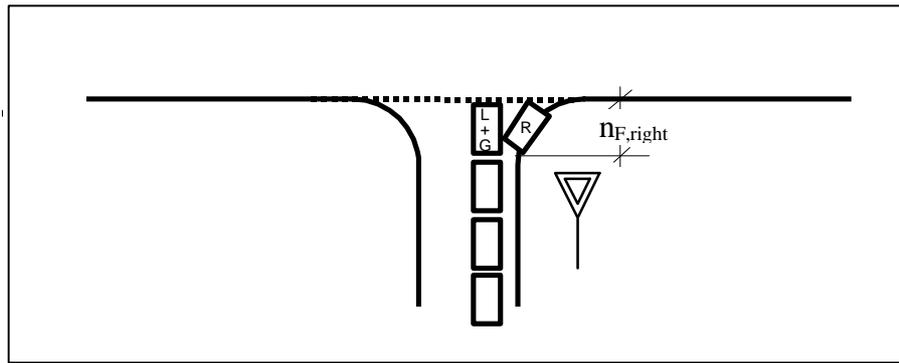


Figure 7. Right flared approach

A special application of Equations (31) and (40) is the so-called flared lanes (Figure 7). For the right flared approach (right-turn stream passes by the left-turn/through stream), the following relationships are valid:

$$n_L = n_T = 0$$

$$n_{LT} = n_R = n_{F,right}$$

Accordingly, one obtains the postulate

$$P_{s,sh} |_{F,right} = x_{sh,max} = [(k_{F,right} \cdot x_L) + (k_{F,right} \cdot x_T)]^{n_{F,right}+1} + (k_{F,right} \cdot x_R)^{n_{F,right}+1} \stackrel{!}{=} 1$$

Solving this equation for $k_{F,right}$, one obtains

$$k_{F,right} = \frac{1}{n_{F,right}^{n_{F,right}+1} \sqrt[n_{F,right}]{(x_L + x_T)^{n_{F,right}+1} + x_R^{n_{F,right}+1}}} \quad (42)$$

and

$$C_{F,right} = k_{F,right} \cdot \sum q_i = \frac{q_L + q_T + q_R}{n_{F,right}^{n_{F,right}+1} \sqrt[n_{F,right}]{(x_L + x_T)^{n_{F,right}+1} + x_R^{n_{F,right}+1}}} \quad (43)$$

For $n_{F,right} = 1$, it becomes

$$C_{F,rechts} |_{n_{F,rechts}=1} = \frac{q_L + q_T + q_R}{1^{1+1} \sqrt[1]{(x_L + x_T)^{1+1} + x_R^{1+1}}} = \frac{q_L + q_T + q_R}{\sqrt[2]{(x_L + x_T)^2 + x_R^2}} \quad (44)$$

Left flared lane at minor approaches (Figure 8)

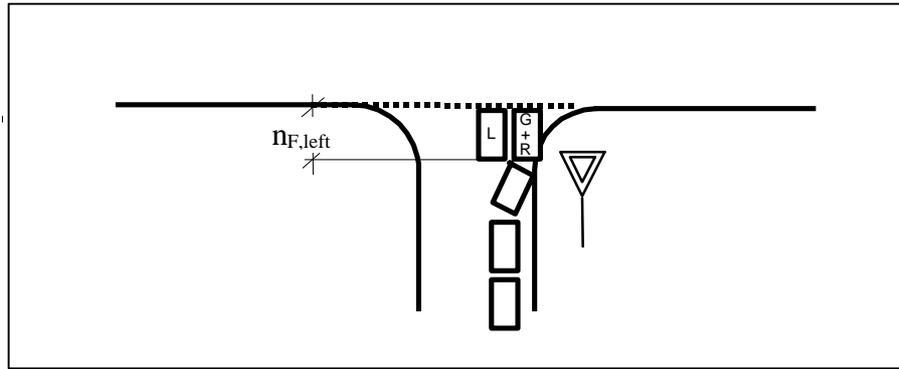


Figure 8. Left flared approach

Analogously, for the left flared approach (left-turn stream passes by the right-turn/through stream, Figure 8), one obtains

$$C_{F,left} = \frac{q_L + q_T + q_R}{n_{F,left}^{n_{F,left}+1} \sqrt[n_{F,left}]{x_L^{n_{F,left}+1} + (x_T + x_R)^{n_{F,left}+1}}} \quad (45)$$

and for $n_{F,left} = 1$

$$C_{F,left} |_{n_{F,left}=1} = \frac{q_L + q_T + q_R}{\sqrt[2]{x_L^2 + (x_T + x_R)^2}} \quad (46)$$

For $n_F = 0$ the Equations (43) and (45) yield

$$C_{F,right} |_{n_{F,right}=0} = C_{F,left} |_{n_{F,left}=0} = \frac{q_L + q_T + q_R}{\sqrt[0+1]{(x_L + x_T)^{0+1} + x_R^{0+1}}} = \frac{q_L + q_T + q_R}{\sqrt[0+1]{x_L^{0+1} + (x_T + x_R)^{0+1}}} = \frac{q_L + q_T + q_R}{x_L + x_T + x_R}$$

Again, one obtains the shared lane formula from Harders (4).

Mixed flared lane at minor approaches (see Figure 9)

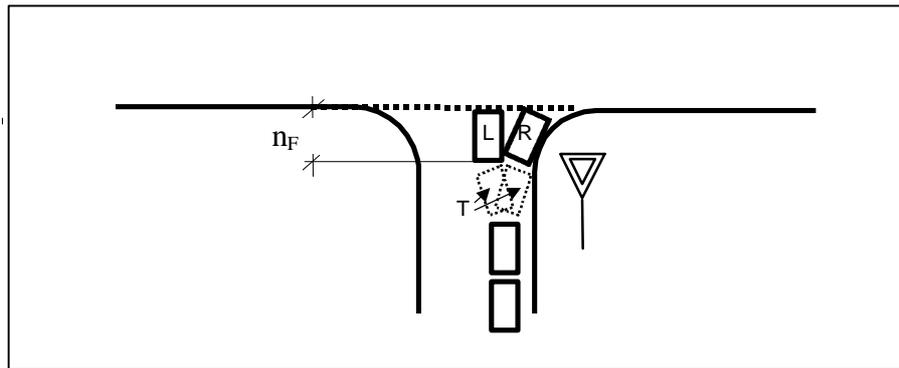


Figure 9. Mixed flared approach

Figures 7 and 8 show the two possibilities a flared approach can be used by vehicles. However, it is not easy to forecast how real-world vehicles would use the flared approach. Here, only the behavior of the drivers in the through vehicles is decisive for the calculation because the right- and left-turn vehicles always pass by each other at a flared approach. The decision of a through driver (whether he passes by a waiting left-turn vehicle or by a waiting right-turn vehicle) determines the configuration of the flared approach. If a through driver passes on the left of a waiting right-turn vehicle, the approach is a right flared approach (because the left-turn vehicles must pass by the waiting right-turn vehicle also). If a through driver passes on the right of a waiting left-turn vehicle, the approach is a left flared approach (because the right-turn vehicles must pass by the waiting left-turn vehicle also). If a through driver arrives while another through vehicle is waiting on the stop line, the approach could also be considered a right flared approach (because in this case only the right-turn vehicles may drive by on the right). As an approximation, one can assume that the probabilities (whether the approach is used as a left or right flared approach) are proportional to the corresponding degree of saturations of the traffic streams. Accordingly, an equation for estimating the capacity of the flared approach with mixed configuration,

which treats the approach both as a left flared approach and a right flared approach, can be represented by (see Figure 9)

$$C_{F,mix} = C_{F,left} \cdot \frac{x_L}{x_L + x_T + x_R} + C_{F,right} \cdot \frac{x_T + x_R}{x_L + x_T + x_R} \quad (47)$$

according to the degree of saturations, respectively.

Inserting Equations (42) and (45) into Equation (47) and setting $n_{F,left} = n_{F,right} = n_F$, one obtains

$$\begin{aligned} C_{F,mix} &= \frac{q_L + q_T + q_R}{n_F^{n_F+1} \sqrt[n_F]{x_L^{n_F+1} + (x_T + x_R)^{n_F+1}}} \cdot \frac{x_L}{x_L + x_T + x_R} + \frac{q_L + q_T + q_R}{n_F^{n_F+1} \sqrt[n_F]{(x_L + x_T)^{n_F+1} + x_R^{n_F+1}}} \cdot \frac{x_T + x_R}{x_L + x_T + x_R} \\ &= \left(\frac{x_L}{n_F^{n_F+1} \sqrt[n_F]{x_L^{n_F+1} + (x_T + x_R)^{n_F+1}}} + \frac{x_T + x_R}{n_F^{n_F+1} \sqrt[n_F]{(x_L + x_T)^{n_F+1} + x_R^{n_F+1}}} \right) \cdot \frac{q_L + q_T + q_R}{x_L + x_T + x_R} \\ &= \left(\frac{x_L}{n_F^{n_F+1} \sqrt[n_F]{x_L^{n_F+1} + (x_T + x_R)^{n_F+1}}} + \frac{x_T + x_R}{n_F^{n_F+1} \sqrt[n_F]{(x_L + x_T)^{n_F+1} + x_R^{n_F+1}}} \right) \cdot C_{n=0} \end{aligned} \quad (48)$$

where

$$C_{n=0} = \frac{q_L + q_T + q_R}{x_L + x_T + x_R}$$

is the capacity of the shared lane for the case $n = 0$ (corresponding to Harders formula (4)).

For $n_F = 1$, the Equation (48) yields

$$\begin{aligned} C_{F,mix} |_{n_F=1} &= \left(\frac{x_L}{\sqrt[2]{x_L^2 + (x_T + x_R)^2}} + \frac{x_T + x_R}{\sqrt[2]{(x_L + x_T)^2 + x_R^2}} \right) \cdot \frac{q_L + q_T + q_R}{x_L + x_T + x_R} \\ &= \left(\frac{x_L}{\sqrt[2]{x_L^2 + (x_T + x_R)^2}} + \frac{x_T + x_R}{\sqrt[2]{(x_L + x_T)^2 + x_R^2}} \right) \cdot C_{n=0} \end{aligned} \quad (49)$$

Figure 10 presents a comparison of capacity increases caused by the flaring of the approach. For this comparison the north bound approach of a standard intersection is calculated. The capacities of the

separate traffic streams are obtained according to the German *Highway Capacity Manual* (2). The traffic flow of this intersection is shown in Figure 11. The calculation yields the parameters $x_L = 0.33$, $x_T = 0.46$, and $x_R = 0.05$ for the subject approach. These parameters characterized qualitatively most of the real traffic conditions at approaches to unsignalized intersections.

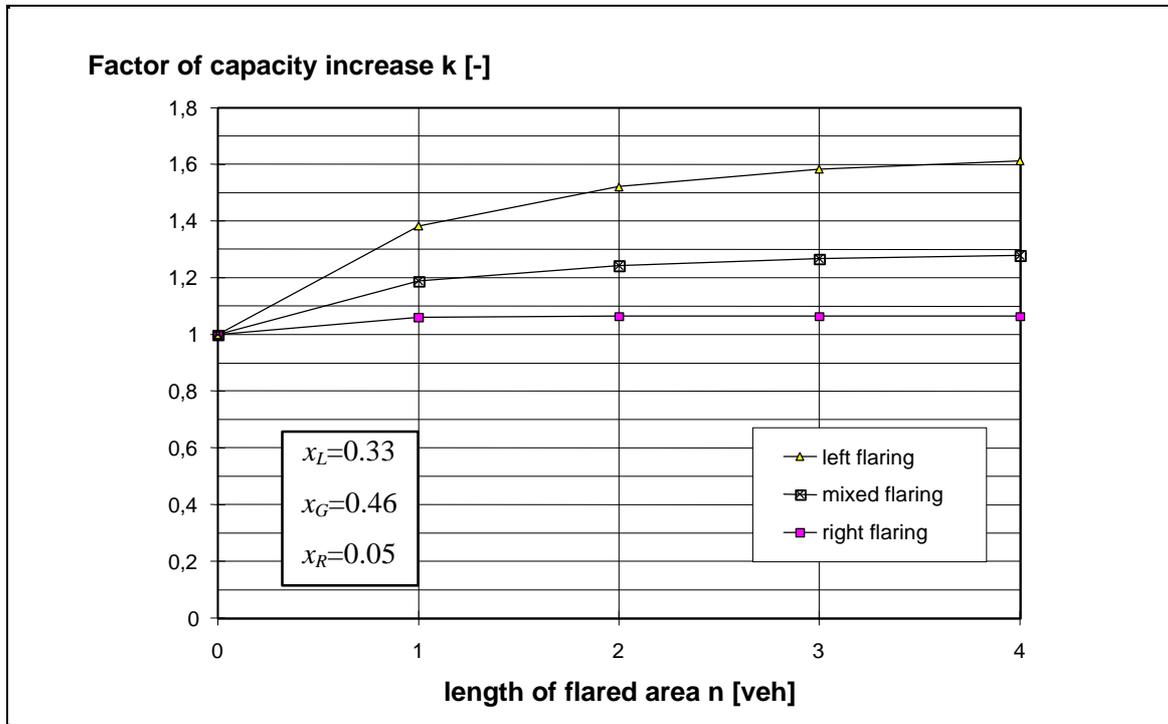


Figure 10. Increases of capacity caused by flaring

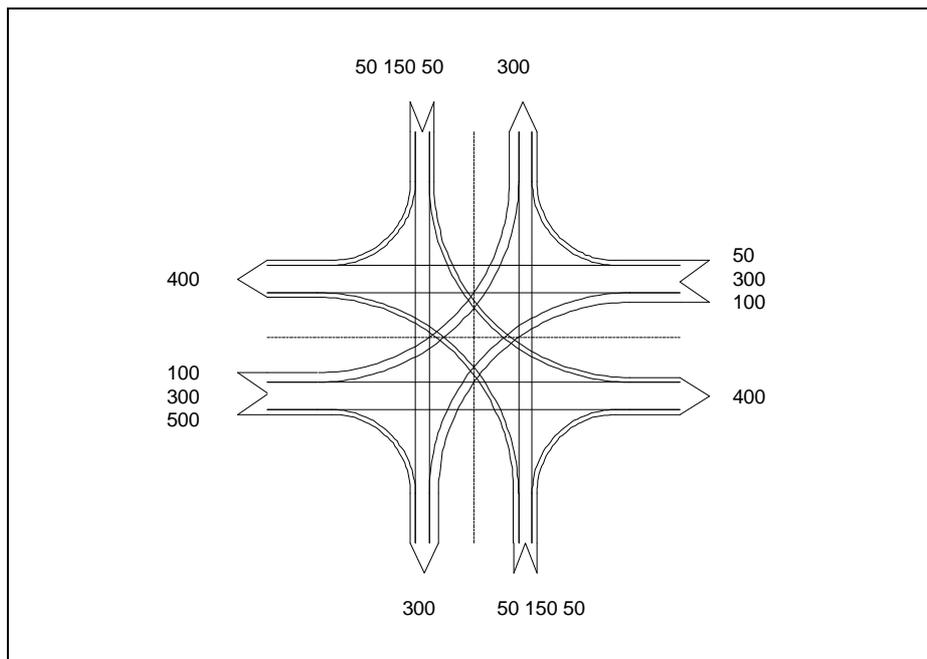


Figure 11. Traffic flow of the test example in [vph]

Clearly one can recognize that the left flaring causes a much greater capacity increase compared with the right flaring. For the left flaring, the example has a capacity increase of 38% at $n_{F,left} = 1$ compared to $n_{F,left} = 0$. For the right flaring, it is barely 6%. The mixed use of the flared area, which is more realistic than the pure left and/or right flaring, delivers approximately 18% increase in capacity. The value of the mixed flaring corresponds very well to the measurements in the technical report from Kyte et al. (8) (see there, Figure 8.7).

TESTING THE THEORY BY SIMULATION

To test the derived theory, different combinations of shared/short lanes are simulated in the style of KNOSIMO (7). Altogether, 95 traffic-flow and lane variations were simulated. The simulation results are presented in Figure 12, together with the theoretical values. The key statistical values of this comparison are assembled in Table 1, which shows that the relationships between both parameters are narrowly correlated. Accordingly, the correctness of the derived theory is verified.

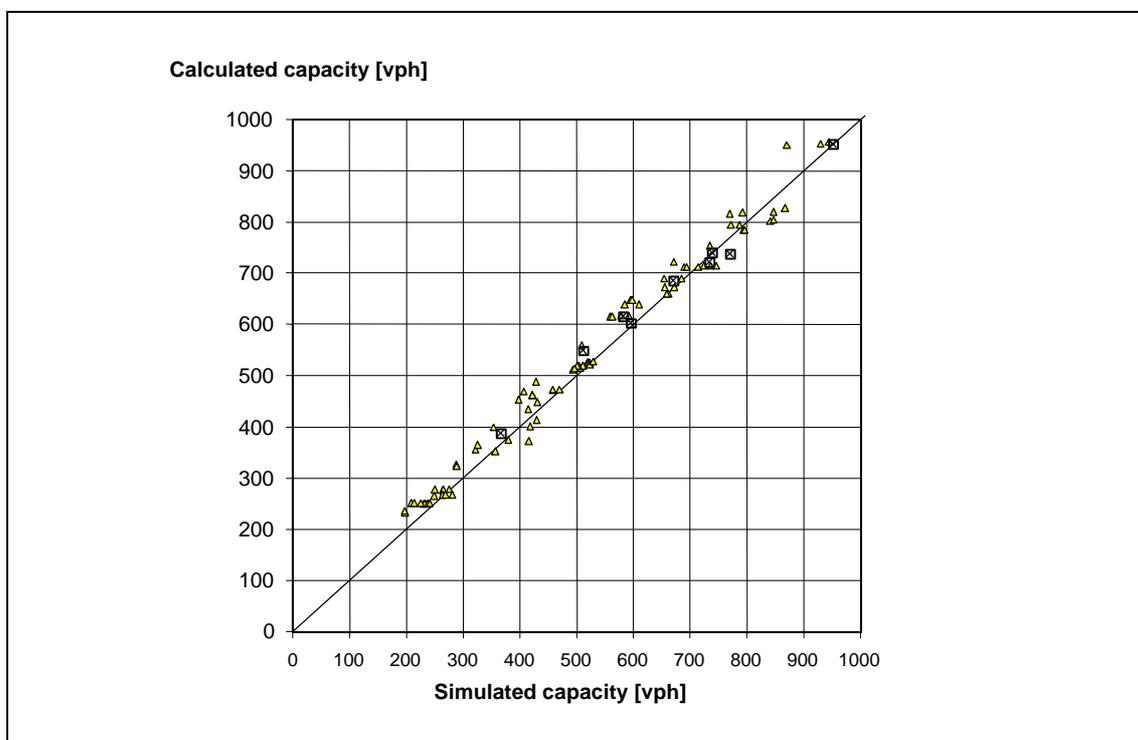


Figure 12. comparison of calculated and simulated capacities

Multiple correlation coefficient [-]	0.993
Certainty [-]	0.985
Adjusted certainty [-]	0.985
Standard errors [vph]	24.42
Observations [-]	95

Table 1. Key statistical values of the comparison

SUMMARY

The theory derived here delivers a general approach for estimating the capacity of shared/short lanes at unsignalized intersections. This theory considers the length of short lanes and fills a gap in the current calculation procedures for unsignalized intersections.

The derivation of this theory presumes that the queuing systems at unsignalized intersections can be approximately considered as M/M/1-queuing systems. This could lead to a minor deviation of the results from reality. The simulation results, however, show that this deviation is negligible and not statistically significant.

For practical applications, Equations (43), (45), and (48) are most important. With these three equations, the capacity of minor approaches at unsignalized intersections with left, right, and mixed flaring can be determined in the simplest and most exact way.

For shared/short lanes with arbitrary lane combinations, a general implicit equation for estimating the capacity is given (Equations (20) and (21)). For the solution of the implicit Equation (20), the Newton iteration method can be used (Equation (22)). With this procedure, all possible shared/short-lane combinations at unsignalized intersections can be determined without large expenses. If a worksheet program is available, one can also use the so-called SOLVER (by Excel) to solve the Equation (20).

As a summary, all possible configurations of shared/short lanes and their solutions are assembled in Table 2. More examples for the theory can be seen in the work from Wu (9).

For estimating the capacity of shared lanes, it is presupposed that the traffic flows of all sub-streams increase proportionally to their original traffic flows. All sub-streams were multiplied by the same factor k . It is also possible, however, to determine the capacity of a particular sub-stream by using fixed traffic flows for all other sub-streams.

The theory should be expanded to intersections with traffic signals

Table 2. Capacity of shared - short lanes C_{sh}

Case	Figure	Equation for $k^{1)}$	Notation
1		<p>Equation (13), iteration procedure.</p> <p>For all $n_i=n$, Equation (11), explicit²⁾.</p>	Generalized case with one level of sub-streams
2		Equation (22), iteration procedure.	Generalized case with two levels of sub-streams
3		<p>Equation (29), iteration procedure.</p> <p>For $n_i=n$, Equation (26), explicit²⁾.</p>	
4		Equation (38), iteration procedure.	
5		Equation (42), explicit ²⁾ .	Right flared lane
6		Equation (45), explicit ²⁾ .	Left flared lane
7		Equation (48), explicit ²⁾ .	Mixed flared lane

¹⁾ $C_{sh} = k \cdot q_{sh} = k \cdot \sum q_i$, ²⁾ C_{sh} is directly available

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Tab.1 - Key statistical values of the comparison

Tab.2 - Capacity of shared/short lanes C_{sh}

Fig.1a - Possible queues at the approaches of unsignalised intersections

Fig.1b - Combination forms of short traffic lanes

Fig.2 - Relationship between a shared lane and its sub - streams

Fig.3 - Relationship between shared lanes and their sub - and sub - sub streams

Fig.4 - Parameters for the type 1 of short traffic lanes

Fig.5 - Parameters for the type 2 of short traffic lanes

Fig.6 - Parameters for the type 3 of short traffic lanes

Fig.7 - Right flared approach

Fig.8 - Left flared approach

Fig.9 - Mixed flared approach

Fig.10 - Traffic flow of the test example

Fig. 11 - Increases of capacity caused by flaring

Fig. 12 - comparison of calculated and simulated capacities