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Modeling Impedance Effects of Left Turners from Major Streets with Shared-Short Lanes at TWSC intersections

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ABSTRACT

At two-way stop controlled intersections without exclusive turning lanes on major streets all three movements (right, through, left turners) must share the same lane. In the HCM 2000, a special formula is provided to deal with this lane configuration. However, this formula cannot consider short lanes on major streets such as the left-turn pockets. This paper presents a new model which takes account for the shared-short lanes and thus delivers more realistic results for estimating the impedance effect through left turners. The new model can also deal with the effect of the so-called Back-of-Queue in reality. The presented theoretical concept can be extended to include also the consequences of a short left turn pocket on major streets and the corresponding capacities of the major streets with shared-short lanes. Some equations are derived for calculating the influence of the length of the left turn pocket on the capacity of minor movements and on the probability of blockage to the through movement due to queued left turn vehicles. Comprehensive simulation studies are conducted in order to confirm the derivation. According to the results of this paper, a general formula which considers both the short-lane and Back-of-Queue is recommended for further applications. This formula will be incorporated in the upcoming new edition of the HCM.

Keywords: Capacity, Unsignalized Intersection, Shared-Short lane

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1. INTRODUCTION

The capacity and quality of traffic flow at unsignalized intersections is estimated based on the gap acceptance theory in most of the existing guidelines, e.g. in the HCM 2000 *(1)*, chapter 17, or in the German HBS 2001 *(2)*, chapter 7. For those methods, however, the treatment of left turners from the major (priority) street (LTPS, eq. 17-16 of the HCM 2000) is not sufficient and they should be improved.

At first, a short theoretical background is given. In this paper the abbreviations and symbols from the HCM 2000 are used. The fundamentals for the critical gap theory are only explained quite shortly. Hereby the reader is referred to *(3)*. This paper is concentrated on the probability of a queue-free state for LTPS movements and the consequences on the through traffic of the major (priority) street (TTPS). On this basis the capacity for minor street movements can be estimated. The solution then is extended towards an expression which estimates the effect of the limited length of a left-turn pocket on major street movements.

2. METHODS FOR SHARED LANES IN THE HCM 2000

Chapter 17 of the HCM 2000 provides the assessment of traffic performance at unsignalized two-way stop controlled (TWSC) intersections. The potential capacity $c_{p,k}$ for each of the twelve movements has to be estimated in advance. In the HCM 2000, this is achieved by Harder's *(4)* formula (eq. 17-3 in the HCM 2000) or with corresponding graphs (exhibit 17-6 and 17-7 in the HCM 2000). Usually, the capacities for priority movements 2, 3, 5, and 6 (cf. FIGURE 1) are predefined to be 1800 veh/h *(2)*. They could also be estimated from field observations *(1)*. These potential capacities $c_{p,k}$ are equal to the movement capacities $c_{m,k}$ if movement k is of rank 2 (eq. 17-4 in the HCM 2000).

The further ranking of priorities is taken into account by using impedance factors to compute the movement capacities $c_{m,k}$ for movement k with a higher rank of priority than 2.

An impedance factor $p_{0,j}$ is the probability that a minor movement j is in a queue-free state (eq. 17-5 in the HCM 2000):

$$p_{0,j} = 1 - g_j = 1 - \frac{v_j}{c_{m,j}} \tag{1}$$

$$c_{m,k} = c_{p,k} \cdot \prod_{j=1}^{J} p_{0,j} \tag{2}$$

where

$p_{0,j}$	=	probability of a queue-free state in minor movement j	[-]
v_j	=	flow rate for minor movement j	[veh/h]

$c_{m\,j}$	=	movement capacity for minor movement j	[veh/h]
g_j	=	degree of saturation for minor movement $j = v_j / c_{m,\,j}$	[-]
$c_{m\,k}$	=	movement capacity for minor movement k	[veh/h]
$c_{p,k}$	=	potential capacity for minor movement k	[veh/h]
k	=	index for minor movements of rank 3 or 4	[-]
j	=	index for minor movements of rank 2 or 3 which have priority over movement k	[-]
J	=	number of minor movements of rank 2 or 3 which have priority over movement k	[-]

The calculation assumes that each movement has its own lane with unlimited length (exclusive lane). The realistic intersection design, however, usually provides only turning lanes of limited extension such as left-turn pockets (cf. FIGURE 2b). Moreover, in many cases there are no turning lanes at all. In those cases, several movements have to share the same lane (cf. FIGURE 2a). Those lanes are called shared lanes. Here, shared lanes on the major street and on the minor approach have a different functionality which leads to different procedures.

For minor street approaches the capacity of a shared lane is estimated by the shared formula from Harders *(3)*, which is also used in the HCM 2000 as eq. 17-15:

$$c_{SH} = \frac{v_i + v_j + v_k}{\frac{v_i}{c_{m,i}} + \frac{v_j}{c_{m,j}} + \frac{v_k}{c_{m,k}}} \tag{3}$$

where

c_{SH}	=	capacity of the shared lane	[veh/h]
v_i, v_j, v_k	=	traffic valume of movements i, j, and k	[veh/h]
$c_{m,i}$, $c_{m,j}$, $c_{m,k}$	=	movement capacities of movement i, j, and k	[veh/h]
i, j, and k	=	7, 8, 9 or 10, 11, 12	

This equation is not exact as demonstrated by *(5)* because the queuing system at TWSC intersections is a so-called M/G2/1 queuing system with two different distributions of service times. The required slight corrections are, however, rather complex and they are of minor importance for the result. Thus, eq. (3) is still quite an adequate solution for application in guidelines.

For a major street with shared lanes we treat one approach containing movements i, j, and k (cf. FIGURE 1). As an example we use $i = 4$, $j = 5$, $k = 6$.

For a shared lane without any turning bay spaces (cf. FIGURE 2a), eq.17-15 of the HCM 2000 defines the probability of queue-free state in movement 4 as

$$p_{0,4}^{*} = 1 - \frac{x_4}{1 - (x_5 + x_6)} \tag{4}$$

where

$p_{0,4}^{*}$	=	probability of queue-free state for the shared lane on the major street	[-]
x_j	=	degree of saturation in movement $j = v_j / c_{m,\,j}$	[-]
x_6	=	0, if movement 6 is operating on a separate lane	

Also this equation goes back to Harders *(3)*. Furthermore, this equation takes account realistically for effect of the so-called Back-of-Queue. The effect of Back-of-Queue is discussed in details later in this paper. This formula cannot consider short lanes on major streets such as left-turn pockets (FIGURE 2b). A new model which takes into account the shared-short lanes and, thus, delivers more realistic results is developed in the following section.

3. IMPEDANCE EFFECTS DERIVED FROM THE PRINCIPLE OF SHARED-SHORT LANES

The total capacity of an approach with short lane configurations can be expressed by an approach developed by Wu, *6*). This function is derived on the background of queuing and probability theory. It takes into account both the stochastic property of the traffic flow and the probability of the lane blockage at the merge point. This is valid both for unsignalized and signalized intersections. However, the model parameters of the approach must be calibrated according to the configurations under consideration. The derivation of the approach is shortly explained below.

At first, a generalized system with *m* sub-movements, which all develop at one merge point from a shared lane (cf. FIGURE 3, point *A*) is considered. A sub-movement *i* is described by the parameters v_i (traffic flow), c_i (capacity) and x_i (degree of saturation). The capacity c_i and the degree of saturation $x_i = v_i/c_i$ are considered under the assumption that there are infinitely many queue places for the subject movement *i*. Accordingly the shared lane has the parameters v_M, c_M and x_M.

For the merge point *A* the following fundamental state condition is valid: *The merge point A is equally occupied from left (shared lane) and from right (all sub-movements) by waiting movements.* That is, the probability that the merge point *A* is occupied on the side of the shared lane is equal to the probability that the point *A* is occupied on the side of the sub-movements. It follows that

$$P_{s,M} = P_{s,1} + P_{s,2} + \ldots + P_{s,i} + \ldots + P_{s,m} = \sum_{i=1}^{m} P_{s,i} \tag{5}$$

where $P_{s,M}$ is the probability that the merge point *A* is by shared lane and $P_{s,i}$ the probability that the point *A* is occupied by the sub-movement *i*.

The probability that the merge point *A* is occupied by a sub-movement is equal to the probability that the queue length in this sub-movement is larger than the number of the queue space (section from the stop line to point *A*), i.e., for sub-movement *i*,

$$P_{s,i} = \Pr(N > n_i) \tag{6}$$

The distribution function of queue lengths in each sub-movement can be represented approximately by the following equation (Wu, *6*):

$$F(n_i) = \Pr(N \le n_i) = 1 - x_i^{1+f(n_i)} \tag{7}$$

with $x_i = v_i / c_i$. The function $f(n_i)$ is a monotonically ascending function of n_i with $f(n_i = 0) = 0$. Thus,

$$P_{s,i} = \Pr(N > n_i) = 1 - F(n_i) = x_i^{1+f(n_i)} \tag{8}$$

For estimating the capacity of the shared lane, the following definition is introduced: *The capacity of the shared lane is the total traffic flow, at which the merge point A on both sides is totally occupied* ($P_{s,M} = x_M = 1$). As a rule, the traffic flow demands v_i (existing or predicted) do not describe the complete saturation of the shared lane. The capacity of the shared lane is generally higher than the sum of v_i (in case of under-saturation by existing v_i). In this case the traffic flow at the subject traffic movement v_M would approach the limit of the capacity, if the v_i-values increase. In general, each v_i-value could have another increase. It is assumed however, that for these increases of existing traffic flows, equal increase factor k are applied to each sub-movement. k is thus that factor, by which all traffic flows on the subject approach has to increase, for reaching the maximal possible traffic flow: the capacity.
Multiplying the saturation degree of all sub-movements by this factor k and postulating

$$P_{s,M} = x_M = \sum_{i=1}^{m} (k \cdot x_i)^{1+f(n_i)} \overset{!}{=} 1 \tag{9}$$

yields the capacity of the subject shared lane:

$$c_M = k \cdot v_M = k \cdot \sum_{i=1}^{m} v_i \tag{10}$$

Accordingly, the real degree of saturation in the shared lane becomes

$$x_{M,real} = \frac{v_M}{c_M} = \frac{1}{k} \tag{11}$$

For the special case with $n_1 = n_2 = ... = n_i = ... = n_m = N_k$, i.e., all sub-movements have the same number of queue space, we get

$$k\,|_{all n_i = N_K} = \frac{1}{\sqrt[1+f(N_k)]{\sum_{i=1}^{m} P_{s,i}}} = \frac{1}{\sqrt[1+f(N_k)]{\sum_{i=1}^{m} x_i^{1+f(N_k)}}} \tag{12}$$

and

$$c_M\,|_{all n_i = N_K} = \frac{\sum_{i=1}^{m} v_i}{\sqrt[1+f(N_k)]{\sum_{i=1}^{m} P_{s,i}}} = \frac{\sum_{i=1}^{m} v_i}{\sqrt[1+f(N_k)]{\sum_{i=1}^{m} x_i^{1+f(N_k)}}} = \frac{1}{\sqrt[1+f(N_k)]{\sum_{i=1}^{m} \left(\frac{a_i}{v_i}\right)_i^{1+f(N_k)}}} \tag{13}$$

a_i is the proportion of flow volume for sub-movement i in the approach. For a configuration with two movements L (left) and T (through), a general form of the approximation function can be expressed as:

$$c_M = \frac{1}{\sqrt[1+f(N_k)]{\left(\frac{a_L}{c_L}\right)^{1+f(N_k)} + \left(\frac{a_T}{c_T}\right)^{1+f(N_k)}}} \tag{14}$$

$$c_{M,L} = c_M \cdot a_L, \quad c_{M,T} = c_M \cdot a_T \tag{15}$$

The function $f(N_k)$ is a monotonically ascending function of N_k with $f(N_k = 0) = 0$. Eq. (14) fulfills all boundary conditions given in Table 1. For example, for $N_k = 0$ (condition 6) we get

$$c_M \mid_{N_k=0} = \frac{1}{\sqrt[1+0]{\left(\frac{a_L}{c_L}\right)^{1+0} + \left(\frac{a_T}{c_T}\right)^{1+0}}} = \frac{1}{\frac{a_L}{c_L} + \frac{a_T}{c_T}} = c_{SH} \tag{16}$$

For $N_k = \infty$ (condition 5) we get

$$c_M \mid_{N_k \to \infty} = \lim_{N_k \to \infty} \frac{1}{\sqrt[1+\infty]{\left(\frac{a_L}{c_L}\right)^{1+\infty} + \left(\frac{a_T}{c_T}\right)^{1+\infty}}} \quad . \tag{17}$$

This equation can be rewritten as

$$c_M \mid_{N_k \to \infty} = \lim_{N_k \to \infty} \frac{1}{\sqrt[1+\infty]{1 + \left(\frac{a_L}{c_L} \Big/ \frac{a_T}{c_T}\right)^{1+\infty}} \cdot \frac{a_L}{c_L}}$$

For $\frac{a_L}{c_L} \Big/ \frac{a_T}{c_T} < 1$,

$$c_M \mid_{N_k \to \infty} = \lim_{N_k \to \infty} \frac{1}{\sqrt[1+\infty]{1 + \left(\frac{a_L}{c_L} \Big/ \frac{a_T}{c_T}\right)^{1+\infty}} \cdot \frac{a_L}{c_L}} = \frac{1}{1 \cdot \frac{a_L}{c_L}} = \frac{c_L}{a_L} < \frac{c_T}{a_T}$$

For $\frac{a_L}{c_L} \Big/ \frac{a_T}{c_T} > 1$,

$$c_M \mid_{N_k \to \infty} = \lim_{N_k \to \infty} \frac{1}{\sqrt[1+\infty]{1 + \left(\frac{a_T}{c_T} \Big/ \frac{a_L}{c_L}\right)^{1+\infty}} \cdot \frac{a_T}{c_T}} = \frac{1}{1 \cdot \frac{a_T}{c_T}} = \frac{c_T}{a_T} < \frac{c_L}{a_L}$$

In the special case with $\frac{a_L}{c_L} \Big/ \frac{a_T}{c_T} = 1$ is $\frac{c_L}{a_L} = \frac{a_T}{c_T}$ we get

$$c_M \mid_{N_k \to \infty} = \lim_{N_k \to \infty} \frac{1}{\sqrt[1+\infty]{1+1} \cdot \frac{a_L}{c_L}} = \frac{1}{1 \cdot \frac{a_L}{c_L}} = \frac{c_L}{a_L} = \frac{c_T}{a_T}$$

Thus, $c_M = \min(c_L/a_L, c_T/a_T)$ for $N_k \to \infty$. That is exactly the boundary conditions 5 in Table 1.

In the HBS 2001 (*2*), a function $f(N_k) = N_k$ (corresponding to an M/M/1 queuing system) is used for TWSC intersections. Thus, eq.(14) yields

$$c_M = \frac{v_L + v_T}{\sqrt[1+N_K]{P_{s,L} + P_{s,T}}} = \frac{v_L + v_T}{\sqrt[1+N_k]{x_L^{1+N_k} + x_T^{1+N_k}}} = \frac{1}{\sqrt[1+N_k]{\left(\frac{a_L}{c_L}\right)^{1+N_k} + \left(\frac{a_T}{c_T}\right)^{1+N_k}}} \tag{18}$$

It is recommended to use this equation for the HCM instead of eqs. 17-34 through 17-36 in order to estimate capacities in flared minor-street approaches at TWSC intersections (cf. FIGURE 4).

To estimate the capacity of a shared/flared right-turn lane, the following equation should be used to compute shared/flared-lane capacity of minor-street approaches:

$$c_{SH,flared} = \frac{v_R + v_{L+T}}{\sqrt[(n_F+1)]{\left(\frac{v_R}{c_R}\right)^{(n_F+1)} + \left(\frac{v_{L+T}}{c_{L+T}}\right)^{(n_F+1)}}} \tag{19}$$

where

$c_{SH,flared}$	=	capacity of the shared/flared lane	[veh/h]
c_R	=	capacity of the right-turn movement	[veh/h]
c_{L+T}	=	capacity of the through and left-turn movements	[veh/h]
v_R	=	right-turn volume	[veh/h]
v_{L+T}	=	through and left-turn volume	[veh/h]
n_F	=	storage places in the flared area (see FIGURE 4)	[veh]

For the special situation of shared lanes without any flaring area (n_F =0) this equation yields (cf. also eq.(3))

$$C_{SH,flared} = \frac{v_R + v_{L+T}}{\frac{v_R}{c_R} + \frac{v_{L+T}}{c_{L+T}}} \tag{20}$$

The procedure for shared/flared-lane of minor-street approaches in the HCM can be significantly improved and simplified.

In a major approach at a TWSC intersection the queue in the though movement has to be understood as a moving convoy with a time headway $t_{\min}$ between the consecutive vehicles, because vehicles in the though movement do not really come to a stop. This convoy does not establish any impedance in sense of the calculation procedures in the HCM. Only such vehicles which arrive directly at the end of a queue in the left turn movement and then really come to a stop will result into additional impedance for the minor movements. Thus, for the major approach, the merge point A will be occupied by the through movement T only if the queues in both movements T and L are larger than the length N_K. That is, with the probabilities $P(n_L > N_k) = x_L^{1+N_k}$ and $P(n_T > N_k) = x_T^{1+N_k}$ we have

$$\begin{aligned} P_{s,T} &= \Pr(N_T > N_k \cup N_L > N_k) \\ &= \Pr(N_T > N_k) \cdot \Pr(N_L > N_k) \\ &= x_T^{1+N_k} \cdot x_L^{1+N_k} = (x_T \cdot x_L)^{1+N_k} \end{aligned} \tag{21}$$

and

$$P_{s,L} = \Pr(N_L > N_k) = x_L^{1+N_k} \tag{22}$$

Thus,

$$\begin{aligned} x_M &= \sqrt[1+N_k]{P_{s,L} + P_{s,T}} \\ &= \sqrt[1+N_k]{x_L^{1+N_k} + (x_L x_T)^{1+N_k}} \\ &= x_L \cdot \sqrt[1+N_k]{1 + x_T^{1+N_k}} \end{aligned} \tag{23}$$

and

$$p_{0,M} = 1 - x_M = 1 - x_L \cdot \sqrt[1+N_k]{1 + x_T^{1+N_k}} \tag{24}$$

This equation satisfies the following boundary conditions which must be fulfilled for a major approach:

$$\begin{aligned} &p_{0,M_{|x_T=0}} = 1 - x_L \\ &p_{0,M_{|N_K=>\infty}} = 1 - x_L \\ &p_{0,M_{|x_L=0}} = 1 \\ &p_{0,M_{N_K=0}} = 1 - x_L \cdot (1 + x_T) = 1 - x_L - x_L \cdot x_T \end{aligned}$$

In reality, there are still more arrivals coming into the end of queue when the queue in front is being discharged. This effect is called Back-of-Queue. The effect of Back-of-Queue is derived as following (cf. also the derivation for the part 1 of delay in the Webster (1958) formula for signalized intersections).

We consider an arbitrary duration of time *t* (cf. FIGURE 5). Without considering the effect of Back-of-Queue, the time occupied at merge point *A* by the movement *T* within a time of duration *t* is

$$x_T^{1+N_k} \cdot t \tag{25}$$

Denoting the additional duration of time (from the end of $x_T^{1+N_K} \cdot t$) that the Back-of-Queue is totally discharged as t^*, the input-output analysis for the time point when the Back-of-Queue is totally discharged yields

$$x_T^{1+N_K} \cdot t \cdot v_T + t^* \cdot v_T = t^* \cdot c_T \tag{26}$$

The solution for t^* is

$$t^* = \frac{x_T^{1+N_K} \cdot x_T \cdot t}{1 - x_T} \tag{27}$$

The total duration t^{**} of occupied time by movement *T* with considering the effect of Back-of-Queue is then

$$
\begin{aligned}
t^{**} &= x_T^{1+N_k} \cdot t + t^* \\
&= x_T^{1+N_k} \cdot t + \frac{x_T^{1+N_k} \cdot x_T \cdot t}{1 - x_T} \\
&= x_T^{1+N_k} \cdot t \cdot (1 + \frac{x_T}{1 - x_T}) \\
&= x_T^{1+N_k} \cdot t \cdot (\frac{1}{1 - x_T})
\end{aligned}
\tag{28}
$$

Thus, in case of taking account the Back-of-Queue, the proportion of time, during which no blockage occurs, is

$$
P_{s,T,BOQ} = \frac{t^{**}}{t} = x_T^{1+N_k} \cdot (\frac{1}{1 - x_T})
$$

That is, if the effect of Back-of-Queue has to be taken account, we must multiply the tern $x_T^{1+N_k}$ with the factor $1/(1 - x_T)$. Thus (cf. eq.(21)),

$$
\begin{aligned}
P_{s,T} &= \frac{\Pr(N_T > N_k)}{1 - x_T} \cdot \Pr(N_L > N_k) = P_{s,T,BOQ} \cdot \Pr(N_L > N_k) \\
&= \frac{x_T^{1+N_k}}{1 - x_T} \cdot x_L^{1+N_k} = \frac{(x_T \cdot x_L)^{1+N_k}}{1 - x_T}
\end{aligned}
\tag{29}
$$

$$
P_{s,L} = \Pr(N_L > N_k) = x_L^{1+N_k} \tag{30}
$$

$$
x_{M,BOQ} = \sqrt[1+N_k]{P_{s,L} + P_{s,T}} = \sqrt[1+N_k]{x_L + \frac{(x_L \cdot x_T)^{1+N_k}}{1 - x_T}} = x_L \cdot \sqrt[1+N_k]{1 + \frac{x_T^{1+N_k}}{1 - x_T}} \tag{31}
$$

and

$$
p_{0,M,BOQ} = 1 - x_M = 1 - x_L \cdot \sqrt[1+N_k]{1 + \frac{x_T^{1+N_k}}{1 - x_T}} \tag{32}
$$

Also this equation satisfies the following boundary conditions necessary for a major approach:

$$
p_{0,M,BOQ|x_T=0} = 1 - x_L
$$

$$
p_{0,M,BOQ|N_k=>\infty} = 1 - x_L
$$

$$
p_{0,M,BOQ|x_L=0} = 1
$$

$$
p_{0,M,BOQ|N_k=0} = 1 - \frac{x_L}{1 - x_T} \quad \text{(cf. Harders 1968 and 17-16 in HCM 2000)}
$$

$$
p_{0,M,BOQ|x_T=1-x_L} = 0 \quad \text{(Corresponding to } x_T + x_L = 1\text{, the approach is saturated)}
$$

In FIGURE 6, a comparison between the probabilities of queue free state with ($p_{0,M,}$ eq.(24)) and without ($p_{0,M,BOQ}$, eq.(32)) Back-of-Queue is illustrated. One can recognise that the

probability with Back-of-Queue is always smaller than the probability without Back-of-Queue.

In case without Back-of-Queue we have the generalized form

$$p_{0,i}{}^* = Max\begin{cases} 1 - x_i \cdot \sqrt[n_L+1]{1 + \left(x_j + x_k\right)^{n_L+1}} \\ 0 \end{cases} \tag{33}$$

And in case with Back-of-Queue

$$p_{0,i}{}^* = Max\begin{cases} 1 - x_i \cdot \sqrt[n_L+1]{1 + \dfrac{\left(x_j + x_k\right)^{n_L+1}}{1 - \left(x_j + x_k\right)}} \\ 0 \end{cases} \tag{34}$$

where

i	=	1 or 4	
j	=	2 or 5	
k	=	3 or 6	
$p_{0,i}{}^*$	=	probability of a queue free state on the shared lane of the major street	[-]
x_i, x_j, x_k	=	degree of saturation for movements i, j, k	[-]

According to the conducted simulation studies earlier, the results for the probability $p_{0,i}{}^*$ of queue-free state are very sensitive to the simulated capacities in movements 4 and 5. The deviations of those probabilities are highly dependent on the deviations of the capacities. On the other side, the conducted, event-oriented simulation cannot reproduce the effect of Back-of-Queue.

In order to cover the deviation of capacities in the simulation and to account for the effect of Back-of-Queue, an additional time-oriented simulation study is conducted with predefined average capacities for movements 4 and 5. The queuing systems in movements 4 and 5 are presumed as M/M/1 queuing systems. That is, both intra-vehicle headways and service times are assumed to be exponentially distributed. That means, the arrivals and the capacities in a given time interval are assumed to be Poison-distributed. The simulation is conducted in a time step of 0.1s. The probability of queue-free state in the major approach (movements 4 and 5) can be extracted directly.

In Table 2, combinations for average demand/capacity values used in the simulation study are illustrated. The probabilities of queue-free state can be obtained both in cases with and without Back-of-Queue. In FIGURE 7, the results without Back-of-Queue are illustrated. FIGURE 8 shows the results with Back-of-Queue.

The results of all conducted simulations indicate that eq. (33) and eq. (34) lead to more realistic results for the probabilities of queue-free state in major approaches with shared movements 4 and 5 under presumed conditions (with or without Back-of-Queue). Because the effect of Bach-of-Queue does exist in the reality, eq. (34) is finally recommended to be incorporated into guidelines (HCM and HBS).

As a conclusion, eq. (34) with consideration of Back-of-Queue is recommended to be used in HCM (or in other guidelines) for estimating the queue-free probabilities in major approaches

in place of eq. 17-16 in the HCM. In addition, from eq. (34) also the blocking probabilities p_i^* in the major approach caused by the left-turn movement can be estimated as following:

$$p_i * = 1 - p_{0,i} * = x_i \cdot \sqrt[n_L+1]{1 + \frac{(x_j + x_k)^{n_L+1}}{1-(x_j + x_k)}} \tag{35}$$

For $n_L = 0$ eq. (34) yields.

$$p_{0,i} * = 1 - x_i \cdot \left(1 + \frac{(x_j + x_k)}{1-(x_j + x_k)}\right) \tag{36}$$

$$p_i * = x_i \cdot \left(1 + \frac{(x_j + x_k)}{1-(x_j + x_k)}\right) \tag{37}$$

These are the queue-free probability in the major approach and the blocking probability that the through movement is blockaded by left turn movement in the shared-lane situation.

The shared/short-lane capacity on major streets can be then computed as

$$c_{SH,short} = \min\left(\frac{v_j + v_{i1} + v_{i2}}{x_M}, s_{i,1+2}\right) = \min\left(\frac{v_j + v_{i1} + v_{i2}}{p_j *}, s_{i,1+2}\right) \tag{38}$$

where

$c_{SH,short}$	=	capacity of the shared/short lane on major streets	[veh/h]
p_j^*	=	probability that there will be queue in the major street shared/short lane (eq.(35))	[-]
	=	$1 - p_{0,j}^*$	
$p_{0,j}^*$	=	probability that there will be no queue in the major street shared/short lane (eq.(34))	[-]
j	=	1, 4 (major-street left-turning vehicular movements)	
$i1$	=	2, 5 (major-street through vehicular movements)	
$i2$	=	3, 6 (major-street right-turning vehicular movements)	
v_j	=	major-street left-turning movement flow rate	[veh/h]
$s_{i,1+2}$	=	$\frac{v_{i1} + v_{i2}}{\frac{v_{i1}}{s_{i1}} + \frac{v_{i2}}{s_{i2}}}$ = combined saturation flow rate for the major-street through movements (this parameter can be measured in the field)	[veh/h]
s_{i2}	=	saturation flow rate for the major-street and right-turn movements (this parameter can be measured in the field);	[veh/h]
v_{i1}	=	major-street through movement flow rate	[veh/h]
v_{i2}	=	major-street right-turn flow rate (or 0 if an exclusive right-turn lane is provided).	[veh/h]

Brilon (*7*) introduced another model dealing with the same problem. However, his model is considered only as a simplification. The model introduced in this paper is a modification to the earlier model from Brilon (*7*). According to the simulation studies, the new model is more accurate than the earlier model (*7*). Thus, it is preferred to use the new model instead of the earlier model (*7*).

4. SUMMERY

This paper provides several theoretical derivations of the shared-short lane formulas to be applied in the case of shared-short lane situations on the major street at TWSC intersections, especially for major streets with shared-short lanes. The developed formulas are recommended for further applications. These formulas could be verified by a series of simulations. In addition, these formulas can be extended to cover also the effects of a short left-turn lane (e.g. a left-turn pocket) on major streets. Those extended formulas fulfill all the restrictions which are typical for the problem.

In the upcoming new edition of the HCM, eq. 17-16 of the HCM 2000 is replaced by the new formula eq. (34). This approach is also incorporated into the new edition of the German HBS *(2)*.

Finally, it should be noted that also these derivations are not an exact mathematical solution to the problem of TWSC intersections. Like the whole gap acceptance theory, also the derivations submitted in this paper are more like an application of rather pragmatic mathematics, since a series of simplifying assumptions are needed to come to a solution ready for use in practice.

5. REFERENCES

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List of Tables and Figures

Table 1 - Necessary boundary conditions for capacities of an approach at TWSC intersections with short-shared lanes

No.	boundary condition	note
1	$c_{M,L} \leq c_L$	a)
2	$c_{M,T} \leq c_T$	a)
3	$c_M = c_L$ for $v_T = 0$	b)
4	$C_M = c_T$ for $v_L = 0$	b)
5	$c_M = \min(c_L \cdot (v_L + v_T)/v_L,\ c_T \cdot (v_L + v_T)/v_T)$ for $N_k \rightarrow \infty$	c)
6	$c_M = c_{SH}$ for $N_k = 0$	d)

[a)] The capacity of a short lane is always smaller than the capacity of an exclusive lane

[b)] The capacity of the approach is equal to the capacity of an exclusive lane if the flow rate of one of both lanes is zero

[c)] The ratio between the flow rates of both lanes remains constant for $N_k \rightarrow \infty$

[d)] The capacity of the approach is equal to the capacity of a shared lane for $N_k = 0$

Table 2 – Demand/capacity combinations for the simulation study

v_5 (veh/h)	700	700	700	900	1000	600
$v_{m,5}$ (veh/h)	1800	1800	1800	1800	1800	1800
v_4 (veh/h)	100	200	300	200	200	300
$c_{m,4}$ (veh/h)	500	500	500	500	500	500

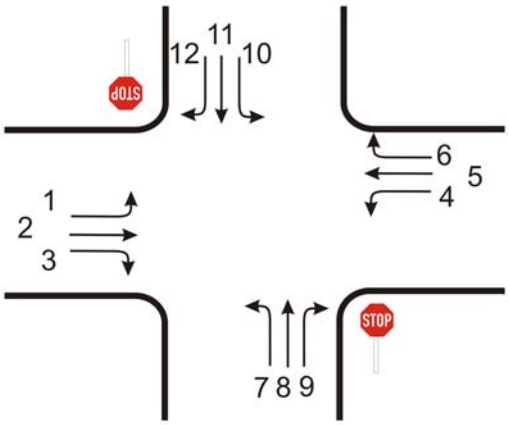
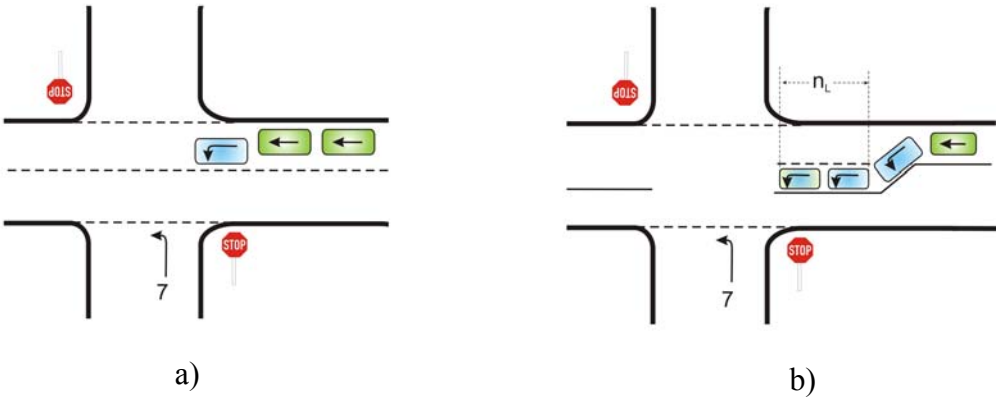


FIGURE 1 - Definition of movements at an unsignalized intersection in the HCM 2000



a) b)

FIGURE 2 - Shared lanes (a) and shared- short lanes (b)

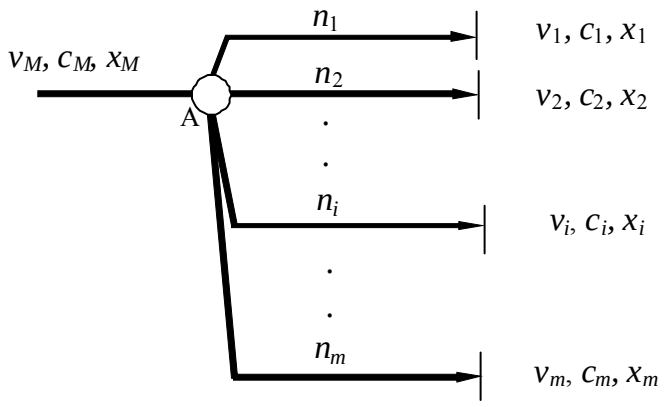


FIGURE 3 - Relationship between a shared lane and its sub - movements

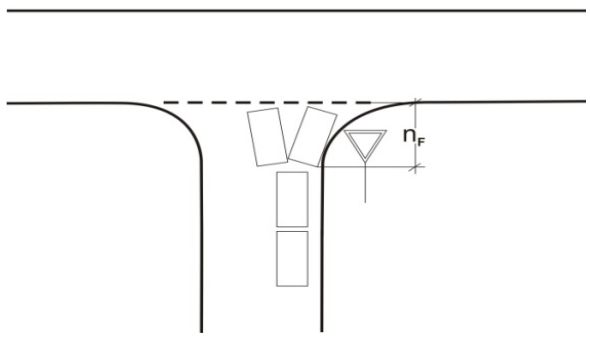


FIGURE 4 - Flared Minor-Street approach

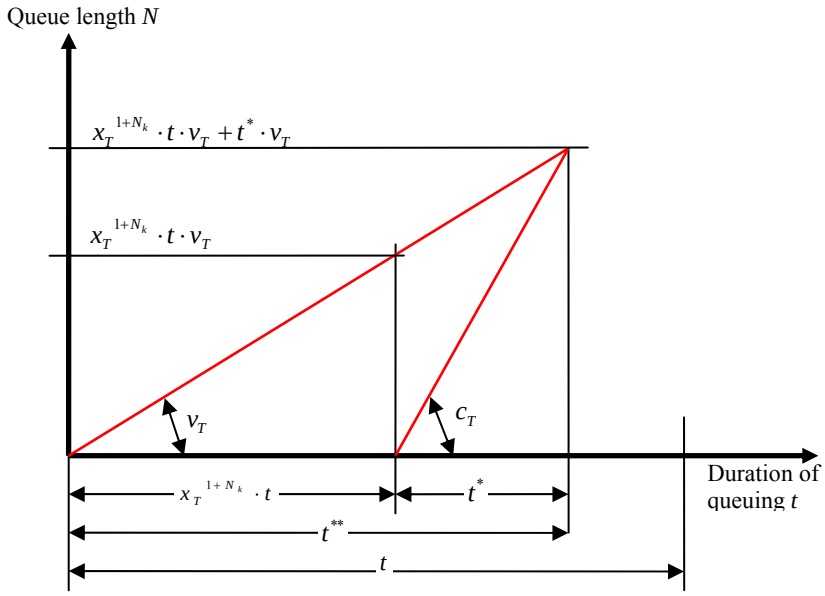


FIGURE 5 – Effect of Back-of-Queue

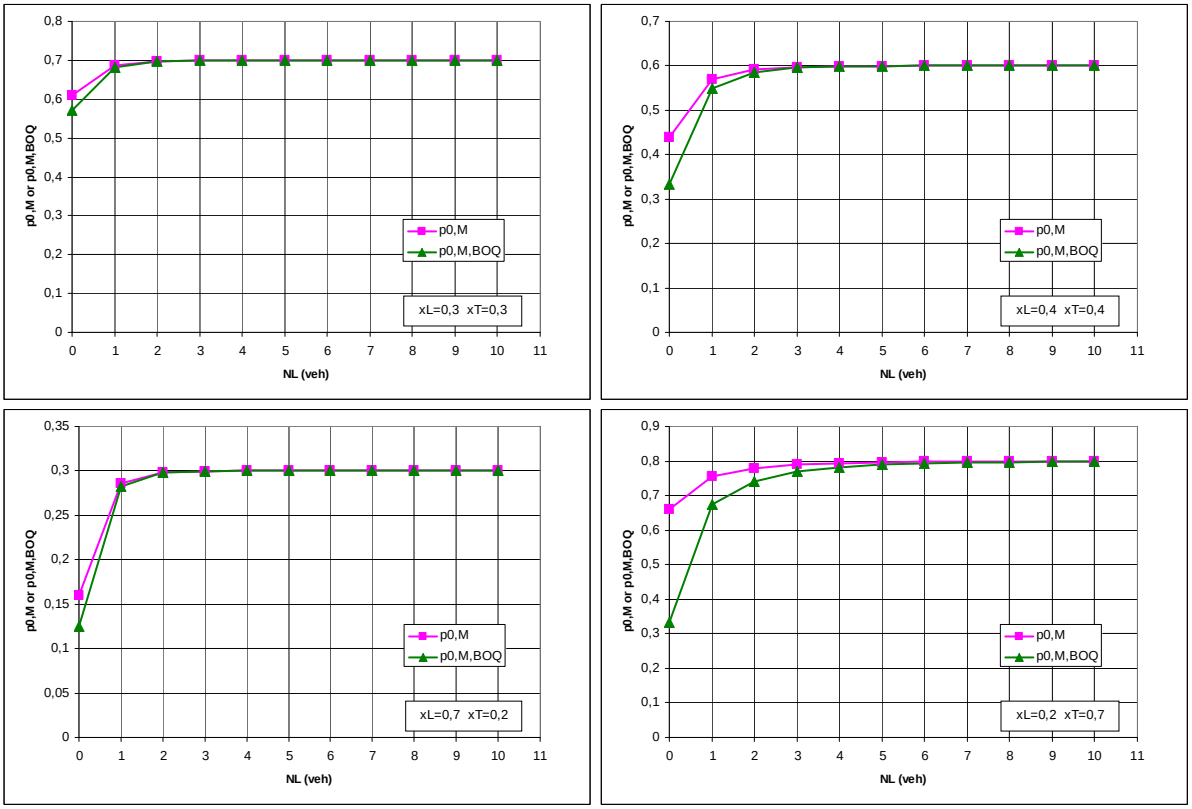


FIGURE 6 - Comparison between the probabilities of queue free-state with ($p_{0,M}$, eq.(24)) and without ($p_{0,M,BOQ}$, eq.(32)) Back-of-Queue

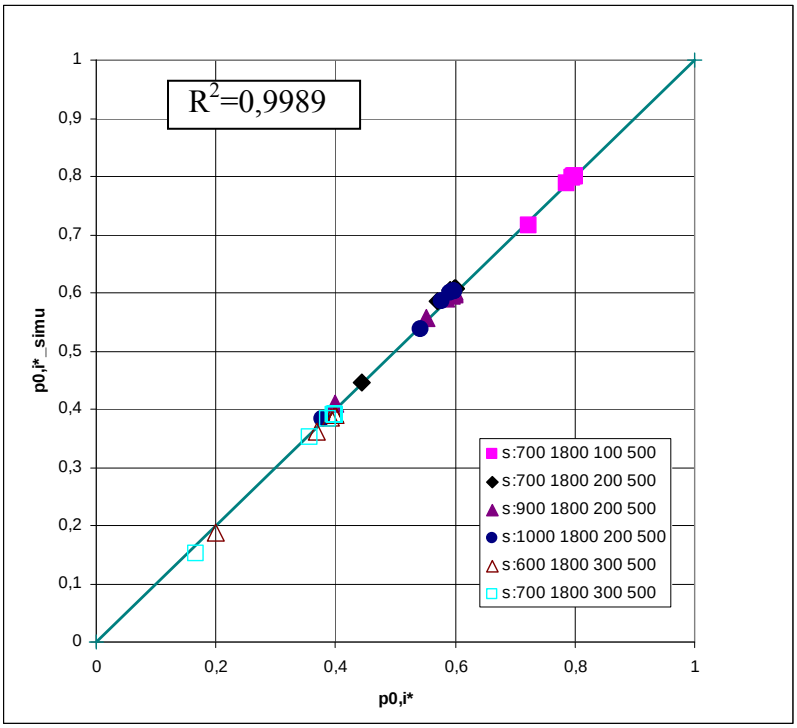


FIGURE 7 - Probabilities of queue-free state $p_{0,i}^*$ in case without considering Back-of-Queue compared to the M/M/1 simulation results for eq. (33)

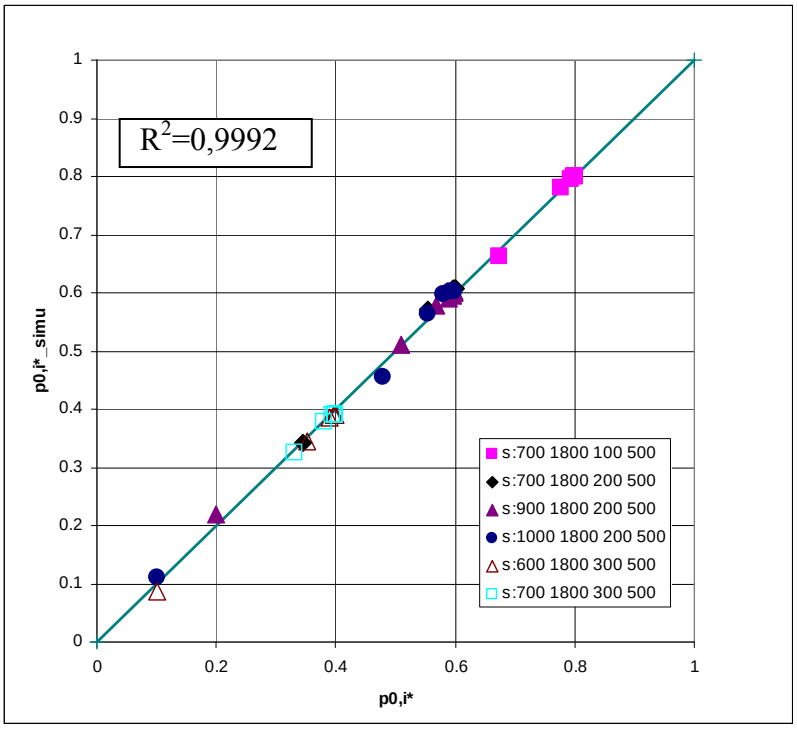


FIGURE 8 - Probabilities of queue-free state $p_{0,i}^{*}$ in case with considering Back-of-Queue compared to the M/M/1 simulation results for eq. (34)