A UNIVERSAL PROCEDURE FOR CAPACITY DETERMINATION AT UNSIGNALIZED INTERSECTIONS

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ABSTRACT

This paper introduces a universal procedure for calculating the capacity at unsignalized intersections. The procedure is based on the idea that the time scale of the major stream can be divided into four regimes according to the relative positions between the vehicles in the major stream: 1) that of free space (no vehicle), 2) that of single vehicle, 3) that of bunching, and 4) that of queuing. The probability of these regimes can be calculated according to the queuing theory. Therefore, the capacity of the minor stream that depends predominantly on the probability of the state that no vehicle blocks the major streams (state of free space) can also be calculated.

The present procedure is derived mathematically using queuing theory. It generalizes all of the known procedures for calculation capacities at unsignalised intersection. The model is calibrated and verified by measurements at roundabouts and by intensive simulations. The results of the present procedure are already incorporated into the 2000 German Highway Capacity Manual.

1 INTRODUCTION

Unsignalized intersections (priority-controlled intersections) are the mostly used type of road junctions in highway transportation systems. The capacity at these intersections is thereby one of the most researched topics in traffic science and engineering. The capacity of a traffic facility describes the maximum possible throughput of the facility under predefined conditions. Starting from the capacity, further traffic parameters which represent traffic quality can be calculated.

At unsignalized intersections, there are traffic streams which have different ranks in the priority hierarchy. Depending on which stream is considered different queuing systems result. For calculating the capacity of these queuing systems different procedures should be used.

The methods for calculating the capacity can basically be divided into two groups: 1) Calculation of the capacity of a simple queuing system with two streams: one major stream and one minor stream and 2) Calculation of the capacity of a comprehensive queuing system with more than two streams of different rank in the priority regulation.

In the group "queuing systems with one major stream and one minor stream", a large variety of calculation methods which yield the corresponding accuracy depending on the assumed traffic conditions exists. Here, one has firstly mathematical solutions that based on the theory of stochastic processes and gap-acceptance. In the group "queuing systems with more than two streams", only one pragmatic procedure exits for practice uses. This procedure was developed in Germany and has also found broad applications in other countries.

In this paper, we concentrate only on the group "queuing systems with one major stream and one minor stream". The group "queuing systems with more than two streams" is discussed elsewhere by the author (Wu, 1998).

In this paper, most of the known procedures in the category "queuing systems with one major stream and one minor stream" are compiled. They are divided according to their properties into groups. The relationships between these procedures are depicted. The available procedures are then extended and generalized to include further parameters. A new procedure which represents a generalization of the procedures for "queuing systems with one major stream and one minor stream" is presented

The new development completes the procedure for the calculation of capacities at unsignalized intersections. It is derived conclusively and it can be applied simply in practice. This procedure has a systematic structure which allows extension to more complicated systems.

In this paper, the following notations and symbols are used:

$L(t(q))$	=	notation for Laplace transform of t at q	
$E(x)$	=	notation for expected value of x	
Pr()	=	notation for probability	
—	=	notation for mean value	
|	=	notation for "under condition"	
C	=	capacity of the minor stream	[veh/s]
C_s	=	capacity of the minor stream in the Free-space state	[veh/s]
$f(t)$	=	distribution intensity of gaps t in the major stream	[-]
$F(t)$	=	distribution function of gaps t in the major stream	[-]
$g(t)$	=	function for the number of vehicles which can depart during t	[veh/s]
$p_{0,S}$	=	probability for the state of Queuing-free	[-]
$p_{0,B}$	=	probability for the state of Bunching-free | Queuing-free	[-]
$p_{0,F}$	=	probability for the state of Vehicle-free | (Bunching-free | Queuing-free)	[-]
q_f	=	$\frac{\varphi \cdot q_p}{1-q_p \cdot \tau}$ = traffic intensity within the portion of free traffic	[veh/s]
q_p	=	traffic intensity in the major stream	[veh/s]
t	=	length of a time headway in major stream (gap)	[s]
t_0	=	$t_g - \frac{t_f}{2}$ = zero-gap	[s]
t_f	=	move-up time	[s]
t_g	=	critical gap	[s]
x	=	saturation degree of the queuing system	[-]
α	=	parameter of the Erlang-distribution	[-]
φ	=	portion of free traffic in the major stream	[-]
τ	=	minimum gap between two vehicles going in succession	[s]
τ_{tf}	=	minimum for t_f	[s]
τ_{tg}	=	minimum for t_g	[s]
τ_τ	=	minimum for τ	[s]

2 QUEUING SYSTEM WITH ONE MAJOR STREAM AND ONE MINOR STREAM.

The queuing system with only one major stream and one minor stream is a so-called M/G2/1 queuing system. For this system, many mathematical approaches have been developed. These approaches have their validity under different predefined traffic conditions:

- free and bunched traffic
- discrete and continuous departure
- fixed and distributed time headways
- consistent and inconsistent driver behavior

For these different conditions, deferent different formulae can be obtained.

A queuing system with two streams which cross themselves (Fig.**1**) is now considered. The major stream has the right of priority and can drive through without stopping at the intersections. The minor stream has to give way to the major stream and stop appropriately. A vehicle from the minor stream can only depart crossing the major stream (or merging into the major stream), when a large time headway (gap) is offered between two vehicles in the major stream. The classic procedure for the determination of capacity is based on the calculation of the distribution of gaps in the major stream and on the calculation of the number of vehicles which can depart during a gap within the major stream. Accordingly, the capacity C of the minor stream is given by

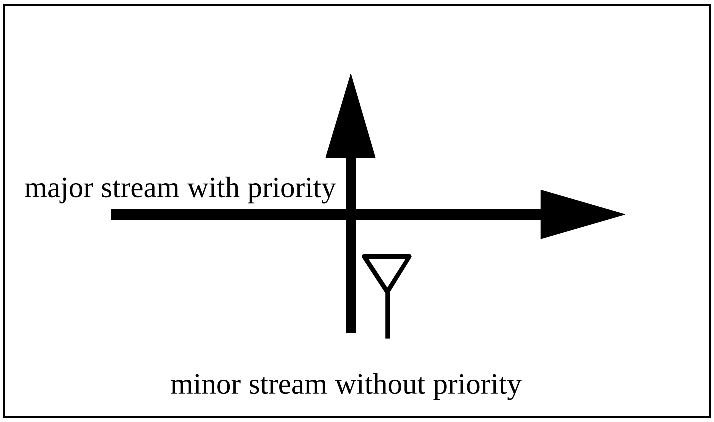

Fig. 1 - System with a major stream and a minor stream

$$C = q_p \cdot \int_0^{\infty} f(t) \cdot g(t) \cdot dt \qquad (1)$$

(cf. Siegloch, 1973). Here, f(t) is the probability density of gaps t in the major stream, g(t) is the function for the number of vehicles which can depart during a gap of the length t, and q_p is the traffic intensity per unit of time in the major stream. Eq.(1) indicates the sum of vehicles departing during all gaps in the major stream: the capacity C in vehicle per unit of time. Depending on which function for f(t) and g(t) is used, different formulae for the determination of capacity C can result.

2.1 Free and bunched traffic flow in the major stream

For choice of functions of the probability density of gaps f(t) two assumptions modeling the traffic flow in the major stream are presupposed:

- free traffic flow in the major stream

Under free traffic flow it is assumed that a vehicle does not influence the vehicles going behind him. Mathematically means: the arrivals of vehicles which go in succession are by chance and absolutely independent of each other; the gaps between two vehicles can also take the value of zero.

- bunched traffic
  Under bunched traffic flow it is assumed that between two vehicles which go in succession a minimum gap has to be held. From this assumption a different distribution of gaps, compared to that for the free traffic flow, can be obtained.

Clearly, these assumptions are only true under predefined conditions.

Under the assumption that the arrivals of vehicles in the major stream are completely coincidental (free), the gaps t between two vehicles are negative-exponentially distributesd. The probability density of gaps t between two vehicles reads

$$f(t) = q_p \cdot e^{-q_p \cdot t} \qquad (2)$$

If the arrivals of vehicles in the major stream are not completely stochastic but depend on the vehicle in the front, then the traffic in the major stream is no more completely free. A vehicle must keep a minimum gap τ to the vehicle in the front and drive in succession. One speaks in this case of bunched traffic. The distribution of gaps in the bunched major stream can be described with the shifted-negative-exponentially distribution. The probability density of the shifted-negative-exponentially distributed gaps t reads:

$$f(t) = q_f \cdot e^{-q_f \cdot (t-\tau)} \qquad (3)$$

with φ = portion of free traffic within the major stream

$q_f = \dfrac{\varphi \cdot q_p}{1 - q_p \cdot \tau}$ = traffic density within the portion of free traffic

τ = minimum gap between two vehicles going in succession

The relationship between q and q_f is given by

$$(1-\varphi) \cdot \tau \cdot q_p + \varphi \cdot (\tau + \frac{1}{q_f}) \cdot q_p = (\tau + \frac{\varphi}{q_f}) \cdot q_p = 1 \qquad (4)$$

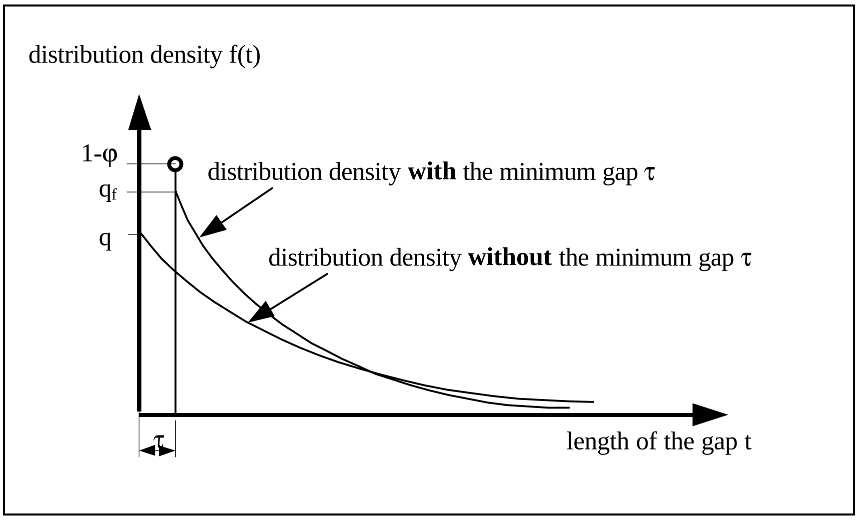

Fig.2 - Probability density of gaps in the major stream f(t)

This equation assumes that the mean length of gap t has within the bunched portion of the traffic the value τ and within the free portion of the traffic the value $\tau + 1/q_f$. The portion of free traffic φ within the major stream describes the portion of the vehicles which go in succession with a gap $t > \tau$. φ depends in general on the traffic intensity q_p in the major stream. In the case of free input, i.e., the up-stream traffic in the major stream is considered as absolutely coincidental, bunched traffic is only caused by compliance with the minimum gap τ. Under the assumption that keeping of a minimum gap τ affects the vehicles within the major stream like a M/D/1 queuing system, Tanner (1962) specified the portion of the free traffic by

$$\varphi = 1 - q_p \cdot \tau \qquad (5)$$

Jacobs (1980) proposed the estimate of the portion of the free traffic as following

$$\varphi = e^{-k \cdot q_p} \qquad (6)$$

In this case, k is a parameter with a value between 4 and 9.

Fig.2 shows a schematic representation of both kinds of the probability density f(t).

2.2 Departure from the minor stream through (or into) the major stream

During choice of functions for the number of departures from the minor stream crossing (or merging into) the major stream two usual models with two different assumptions are available for the function g(t):

- discrete departure from the minor stream
- continuous departure from the minor stream

For the discrete departure, it is assumed that within the major stream the gap t with the length $t_g \leq t \leq t_g + t_f$ enables the departure of one vehicle, the gap t with the length $t_g + t_f \leq t \leq t_g + 2 \cdot t_f$ enables the departure of two vehicles, the gap t with the length $t_g + 2 \cdot t_f \leq t \leq t_g + 3 \cdot t_f$ enables the departure of three vehicles and so on (cf. Fig.3 and Fig.4). The discrete departure function g(t) reads (cf. Harders, 1976)

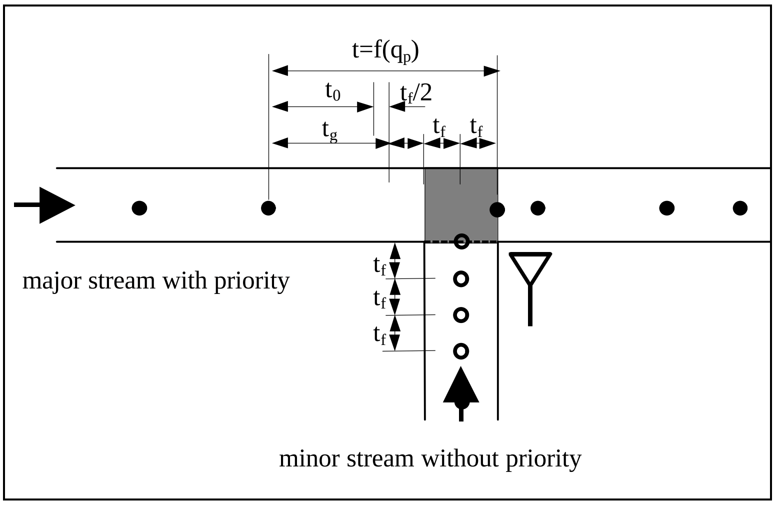

Fig. 3 - Departure mechanism with a free major stream

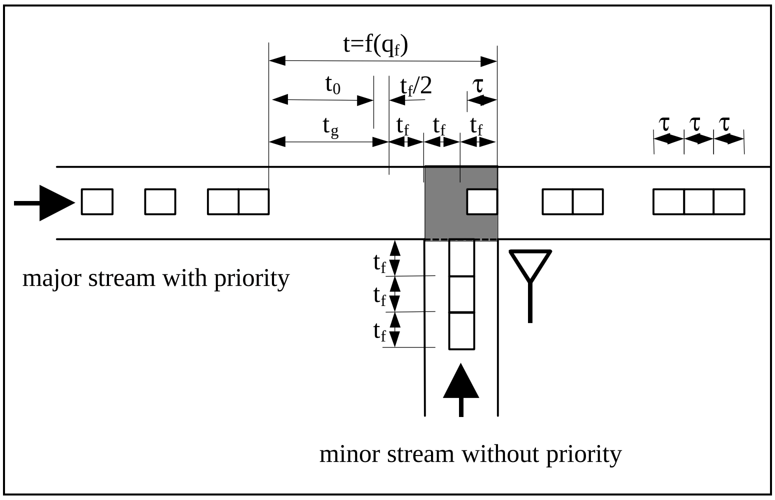

Fig. 4 - Departure mechanism with a bunched major stream

$$g(t) = \begin{cases} \text{int}\left(\frac{t - t_g}{t_f}\right) & \text{for } t \geq t_g \\ 0 & \text{for } t < t_g \end{cases} \tag{7}$$

with t_g = critical gap

t_f = move-up time

The corresponding density function for the departure reads

$$g'(t) = \begin{cases} 1 & \text{for } t \geq t_g \text{ and } \text{mod}\left(\frac{t - t_g}{t_f}\right) = 0 \\ 0 & \text{for } t < t_g \end{cases} \tag{8}$$

For the continuous departure the function g(t) reads (cf. Siegloch, 1973)

$$g(t) = \begin{cases} \frac{t - t_0}{t_f} & \text{for } t \geq t_0 \\ 0 & \text{for } t < t_0 \end{cases} \tag{9}$$

and

$$g'(t) = \begin{cases} \frac{1}{t_f} & \text{for } t \geq t_0 \\ 0 & \text{for } t < t_0 \end{cases} \tag{10}$$

with $t_0 = t_g - \frac{t_f}{2}$

Fig.5 shows the shapes of the two functions g(t) mentioned above.

In the following, separate formulae for the determination of capacity with discrete and continuous departures from the minor stream are derived. Under specific conditions, formulae for discrete departure and continuous departure (cf. section 2.5) can be converted pair-wise into each other.

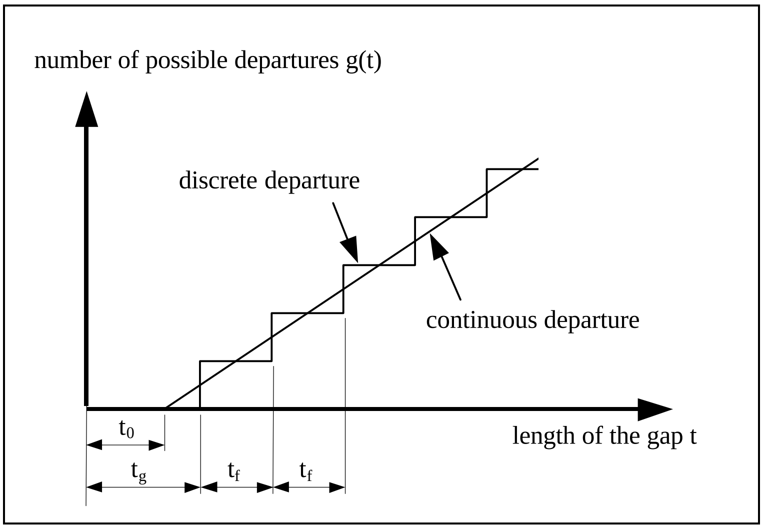

Fig. 5 – Shape of the function for the departure g(t)

2.3 Capacity of systems with one major stream and one minor stream

Setting eqs.(2), (7), and (9) into eq.(1) or setting eqs.(3), (7), and (9) into eq.(1) one obtains different basic formulae for the determination of capacity of systems with one major stream and one minor stream:

- for discrete departure under free traffic (formula of Harders, 1976)

$$C_{free} = q_p \cdot \frac{e^{-q_p \cdot t_g}}{1 - e^{-q_p \cdot t_f}} \tag{11}$$

- for continuous departure under free traffic (formula of Siegloch, 1973)

$$C_{free} = \frac{1}{t_f} \cdot e^{-q_p \cdot t_0} \tag{12}$$

- for discrete departure under bunched traffic (formula of Plank *et al.*, 1984)

$$\begin{aligned} C_{bunch} &= (1 - q_p \cdot \tau) \cdot \frac{q_f \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \\ &= \varphi \cdot \frac{q_p \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \end{aligned} \tag{13}$$

- and for continuous departure under bunched traffic (formula of Jacobs, 1980)

$$C_{bunch} = \frac{1 - q_p \cdot \tau}{t_f} \cdot e^{-q_f \cdot (t_0 - \tau)} \tag{14}$$

With $\varphi = 1 - q_p \cdot \tau$ (cf. Tanner, 1962)

$$q_f = \frac{\varphi \cdot q_p}{1 - q_p \cdot \tau} = \frac{(1 - q_p \cdot \tau) \cdot q_p}{1 - q_p \cdot \tau} = q_p$$

and correspondingly

$$C_{bunch} = (1 - q_p \cdot \tau) \cdot \frac{q_p \cdot e^{-q_p \cdot (t_g - \tau)}}{1 - e^{-q_p \cdot t_f}} \tag{15}$$

This it is exactly the capacity formula of Tanner (1962).

2.4 Consideration of the distributions of t_g, t_f, and τ

In section 2.3, formulae were derived for fixed parameters t_g, t_f, and τ. In this section, new formulae which consider the distribution of t_g, t_f, τ are presented.

As a assumption, the probability density $f(t_g)$, $f(t_f)$, and $f(\tau)$ for t_g, t_f, and τ can in general be described by an Erlang-function. An Erlang distribution has the density function:

$$erl(t_x) = \frac{\lambda}{(\alpha_{t_x} - 1)!} \cdot (\lambda \cdot t_x)^{\alpha_{t_x} - 1} \cdot e^{-\lambda \cdot t_x} \tag{16}$$

with $\lambda = \frac{\alpha_{t_x}}{\bar{t}_x}$

$\bar{t}_x$ = mean value of t_x

α_{t_x} =parameter of the Erlang-distribution for t_x

For the derivation of the formulae for the determination of capacity with discrete departure under bunched traffic regarding the distribution of t_g, t_f, and τ the result from Heidemann and Wegmann (1997) is used as a initial approach.

One distinguishes another two cases according to the departure behavior during the derivation by choice of a gap: a) inconsistent and b) consistent. In the case of the consistent departure behavior, one assumes that a driver by choice of a gap makes his decision every time, independently of the length of the gaps that he refused before. That is, a driver can accept a gap that is shorter than some gaps he refused before. On the other hand, in the case of the consistent departure behavior one assumes that a driver may accept only a gap that is larger than all gaps he refused before.

In the inconsistent case it is valid according to Heidemann and Wegmann (1997) for

- bunched traffic with a minimum gap τ and a portion of bunched traffic 1- φ,
- discrete departure and
- exponentially distributed gaps t within the portion of free traffic φ:

$$C = \frac{q_f}{1+q_f \cdot \overline{B}} \cdot \frac{L(t_g'(q_f))}{1-L(t_f(q_f))} = \frac{q_f}{1+q_f \cdot \overline{B}} \cdot \frac{L(t_g(q_f)) \cdot L(\tau(-q_f))}{1-L(t_f(q_f))} \tag{17}$$

with $$\overline{B} = \frac{\overline{\tau}}{\varphi} \tag{18}$$

$$t_g' = t_g - \tau \tag{19}$$

and $L(t_g'(q_f))$ = Laplace transform of t_g' at q_f

= $L(t_g(q_f)) \cdot L(\tau(-q_f))$

$L(t_g(q_f))$ = Laplace transform of t_g at q_f

$L(t_f(q_f))$ = Laplace transform of t_f at q_f

$L(\tau(-q_f))$ = Laplace transform of τ at $-q_f$

The eq. (17) can be rewritten as

$$C = (1-q_p \cdot \overline{\tau}) \cdot q_f \cdot \frac{L(t_g'(q_f))}{1-L(t_f(q_f))} = (1-q_p \cdot \overline{\tau}) \cdot q_f \cdot \frac{L(t_g(q_f)) \cdot L(\tau(-q_f))}{1-L(t_f(q_f))} \tag{20}$$

Because the Laplace transform L(t(q)) has always a larger value with distributed t(q) than with fixed t(q), it is apparent according to eq.(17) that in the inconsistent case either the distribution of critical gaps t_g or the distribution of move-up times t_f or the distribution of minimum gaps τ increase the capacity C.

Heidemann and Wegmann (1997) recommended in the consistent case for the conditions above

$$C = \frac{q_f}{1+q_f \cdot \overline{B}} \cdot \frac{1}{\left(1-L(t_f(q_f))\right) \cdot L(t_g'(-q_f))} = (1-q_p \cdot \overline{\tau}) \cdot q_f \cdot \frac{1}{\left(1-L(t_f(q_f))\right) \cdot L(t_g(-q_f)) \cdot L(\tau(q_f))} \tag{21}$$

Therefore, the capacity C is reduced by distributed critical gaps t_g and minimum gaps τ .

According to eq.(16), the Laplace transform of the Erlang-distributed gaps t at q with the probability density $f(t_x)$ is given by

$$L(t_x(q)) = \left(\frac{q \cdot \bar{t}_x}{\alpha_{t_x}} + 1\right)^{-\alpha_{t_x}} \tag{22}$$

Substituting eq.(22) with the corresponding parameters t_g, α_{t_g}, $\bar{t}_f$, α_{t_f}, $\bar{\tau}$, and α_τ into the eq.(20), one obtains

- for the inconsistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot q_f \cdot \frac{\left(\frac{q_f \cdot \bar{t}_g}{\alpha_{t_g}} + 1\right)^{-\alpha_{t_g}} \cdot \left(\frac{-q_f \cdot \bar{\tau}}{\alpha_\tau} + 1\right)^{-\alpha_\tau}}{1 - \left(\frac{q_f \cdot \bar{t}_f}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}}} \tag{23}$$

- and for the consistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot q_f \cdot \frac{\left(\frac{-q_f \cdot \bar{t}_g}{\alpha_{t_g}} + 1\right)^{\alpha_{t_g}} \cdot \left(\frac{q_f \cdot \bar{\tau}}{\alpha_\tau} + 1\right)^{\alpha_\tau}}{1 - \left(\frac{q_f \cdot \bar{t}_f}{\alpha_{t_f}} + 1\right)^{-\alpha_{t_f}}} \tag{24}$$

One obtains analogously for continuous departure with inconsistent behavior

$$\begin{aligned} C &= \frac{1}{1 + q_f \cdot \bar{\bar{B}}} \cdot \frac{L(t_0'(q_f))}{\bar{t}_f} \\ &= (1 - q_p \cdot \bar{\tau}) \cdot \frac{L(t_0(q_f)) \cdot L(\tau(-q_f))}{\bar{t}_f} \end{aligned} \tag{25}$$

and with consistent behavior

$$\begin{aligned} C &= \frac{1}{1 + q_f \cdot \bar{\bar{B}}} \cdot \frac{1}{\bar{t}_f \cdot L(t_0'(q_f))} \\ &= (1 - q_p \cdot \bar{\tau}) \cdot \frac{1}{\bar{t}_f \cdot L(t_0(-q_f)) \cdot L(\tau(q_f))} \end{aligned} \tag{26}$$

with $t_0' = t_0 - \tau$

Here, the distribution of the move-up times t_f has no influence on the capacity. One obtains subsequently for

- continuous departure from the minor stream,
- Erlang-distributed gaps t within the major stream,
- Erlang-distributed zero-gaps t_0,
- arbitrarily distributed move-up times t_f, and
- Erlang-distributed minimum gaps τ in the major stream

in the inconsistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot \frac{e^{-q_f \cdot \tau_{t_0}}}{\bar{t}_f} \cdot \left(\frac{q_f \cdot (\bar{t}_0 - \tau_{t_0})}{\alpha_{t_0}} + 1 \right)^{-\alpha_{t_0}} \qquad (27)$$

and in the consistent case

$$C_{bunch} = (1 - q_p \cdot \bar{\tau}) \cdot \frac{e^{-q_f \cdot \tau_{t_0}}}{\bar{t}_f} \cdot \left(\frac{-q_f \cdot (\bar{t}_0 - \tau_{t_0})}{\alpha_{t_0}} + 1 \right)^{\alpha_{t_0}} \qquad (28)$$

In the real world the departure behavior cannot be found exactly. It is in general assumed that most drivers behave rather inconsistently. The effect of the distribution of critical gaps and move-up times mutually neutralize themselves. If the drivers behave with 50% consistently and 50% inconsistently, the effect is almost missing (cf. Wu, 1997a). Therefore, one can neglect the deference between the consistent and inconsistent behavior for practical uses.

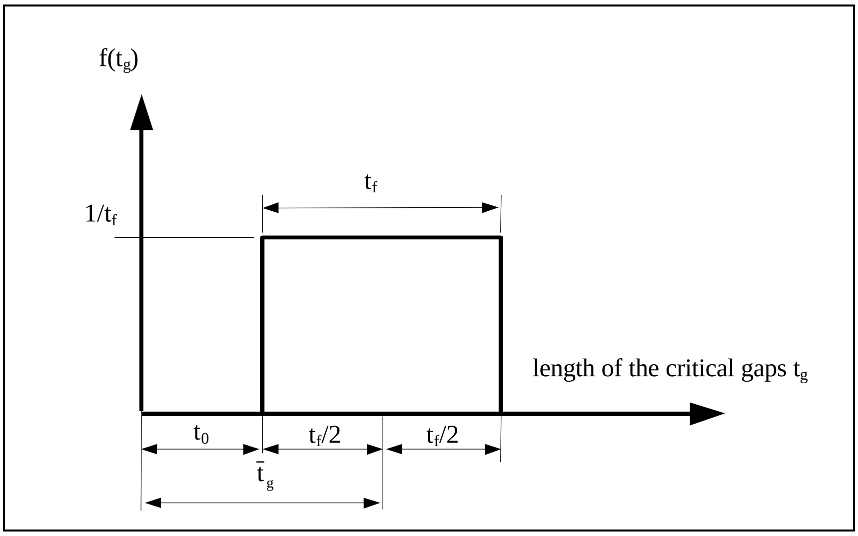

Fig. 6 - Distribution of the critical gaps t_g for the continuous departure

2.5 Relationship between the discrete and the continuous departure

Using a uniform distribution for the critical gaps t_g over the range $\{\bar{t}_g - t_f/2,\ \bar{t}_g + t_f/2\}$, the capacity formula for the continuous departure can be transformed into the capacity formula for the discrete departure. That is, the function of probability density for the critical gaps t_g

$$f(t_g) = \begin{cases} 0 & \text{for } t_g < \bar{t}_g - t_f/2 \\ \frac{1}{t_f} & \text{for } \bar{t}_g - t_f/2 \le t_g \ge \bar{t}_g + t_f/2 \\ 0 & \text{for } t_g > \bar{t}_g + t_f/2 \end{cases}$$

$$= \begin{cases} 0 & \text{for } t_g < t_0 \\ \frac{1}{t_f} & \text{for } t_0 \le t_g \le t_0 + t_f \\ 0 & \text{for } t_g > t_0 + t_f \end{cases} \qquad (29)$$

with $\bar{t}_g$ = mean value of the critical gap t_g

is assumed (cf.Fig.6).

Integrating the eq.(8) piecewise over the eq.(29), one receives

$$g(t) = \begin{cases} 0 & \text{for } t < t_0 \\ \int_{t_0}^{t} g'(t) \cdot f(t_g) \cdot dt_g + 0 & \text{for } t_0 \le t < t_0 + t_f \\ \int_{t_0+t_f}^{t} g'(t) \cdot f(t_g) \cdot dt_g + \int_{t_0}^{t_0+t_f} g'(t) \cdot f(t_g) \cdot dt_g + 0 & \text{for } t_0 + t_f \le t < t_0 + 2t_f \\ . & . \\ . & . \end{cases}$$

$$= \begin{cases} 0 & \text{for } t < t_0 \\ \dfrac{t - t_0}{t_f} & \text{for } t \ge t_0 \end{cases}$$

Which is exactly the eq.(9).

Using in the eq.(11) the distributes critical gaps t_g according to the eq.(29), one obtains then

$$\begin{aligned} C &= \int_0^{\infty} q_p \cdot \frac{e^{-q_p \cdot t_g}}{1 - e^{-q_p \cdot t_f}} \cdot f(t_g) \cdot dt_g \\ &= \frac{1}{t_f} \cdot e^{-q_p \cdot t_0} \end{aligned} \tag{30}$$

Which is exactly the eq.(12).

Also the eq.(17) can be transformed into the eq. (25). Rewriting eq.(17) into

$$C = (1 - q_p \cdot \bar{\tau}) \cdot \frac{q_f}{1 - L(t_f(q_f))} \cdot L(t_0(q_f)) \cdot L(\Delta t_0(q_f)) \cdot L(\tau(-q_f)) \tag{31}$$

with $\quad t_0 + \Delta t_0 = t_g$

and setting

$$\begin{aligned} L(\Delta t_0(q_f)) &= \frac{1 - L(t_f(q_f))}{q_f \cdot \bar{t}_f} \\ &= \frac{1}{\bar{t}_f} \cdot \left(\frac{1}{q_f} - \frac{L(t_f(q_f))}{q_f} \right) \end{aligned} \tag{32}$$

one obtains

$$C = \frac{(1 - q_p \cdot \bar{\tau}) \cdot}{\bar{t}_f} \cdot L(t_0(q_f)) \cdot L(\tau(-q_f))$$

Which is exactly the capacity formula for continuous departure (eq.(25)) under the same condition.

The eq.(32) leads to the following distribution function for the gaps Δt_0

$$f(\Delta t_0) = \frac{1 - F_{t_f}(\Delta t_0)}{\bar{t}_f} \tag{33}$$

or

$$F(\Delta t_0) = \frac{1}{\bar{t}_f} \cdot \int_0^{\Delta t_0} \left(1 - F_{t_f}(t)\right) \cdot dt \tag{34}$$

That is, the eq.(17) turns into the eq.(25) (note, this is only valid for exponentially distributed gaps within the portion of free traffic), if between the distribution of critical gaps t_g and the distribution of zero-gaps t_0 the relationships

$$t_g = t_0 + \Delta t_0$$

and

$$f(t_g) = f(t_0) \otimes f(\Delta t_0) \tag{35}$$

or

$$L(t_g(q_f)) = L(t_0(q_f)) \cdot L(\Delta t_0(q_f)) \tag{36}$$

state. The probability density $f(\Delta t_0)$ is described by eq.(33). It is in turn a function of the distribution of the move-up times t_f. The mean value of Δt_0 reads according to eq.(32)

$$\begin{aligned} \Delta\bar{t}_0 &= E(\Delta t_0) = (-1) \cdot L(\Delta t_0(q_f))'_{|q_f=0} \\ &= \frac{1}{\bar{t}_f} \cdot \left(\frac{(1 - L(t_f(q_f)))}{q_f^{\ 2}} + \frac{L(t_f(q_f))'}{q_f} \right)_{|q_f=0} \\ &= \frac{1}{\bar{t}_f} \cdot \left(\frac{(1 - L(t_f(q_f))) + q_f \cdot L(t_f(q_f))'}{q_f^{\ 2}} \right)_{|q_f=0} \end{aligned}$$

Using the rule of L'hospital, one obtains

$$\begin{aligned} \Delta\bar{t}_0 &= \left(\frac{L(t_f(q_f))''}{2 \cdot \bar{t}_f} \right)_{|q_f=0} \\ &= \frac{\bar{t}_f}{2} + \frac{\sigma_{t_f}^2}{2 \cdot \bar{t}_f} \end{aligned} \tag{37}$$

The eq.(29) is a special case of eq.(33). Setting in the eq.(33) t_f = const., i.e., $\sigma_{t_f}^2 = 0$ and

$$F_{t_f}(t) = \begin{cases} 0 & \text{for } t < t_f \\ 1 & \text{for } t \geq t_f \end{cases}$$

one obtains then

$$f(\Delta t_0) = \begin{cases} \dfrac{1}{t_f} & \text{for } \Delta t_0 < t_f \\ 0 & \text{for } \Delta t_0 \geq t_f \end{cases}$$

with $\Delta t_0 = t_f / 2$

Which is exactly the eq.(29).

3 DETERMINATION OF CAPACITY ACCORDING TO STATES IN THE MAJOR STREAM

In order to be able to extend the capacity formulae to systems with more than one major streams, in this section the capacity in systems with a major stream and a minor stream is considered in another view of point. The resulting formulae correspond to the formulae in the last section. From this point of view, a transfer of the results from systems with one major stream onto systems with more than one major streams is entirely possible.

3.1 Systems with continuous departure

First, one concentrates only on a system with continuous departure with arbitrarily distributed t_g, t_f (therefor also arbitrarily distributed t_0) and τ.

On a time axis one can distinguish periods with: queuing, bunching single vehicle and no vehicle. One can simply sum all small periods with queuing into one large queuing period without the total length of queuing on the time axis is affected. Similarly, one can also sum the small periods of bunching and single vehicle into large periods. Accordingly, the traffic flow in the major streams can be divided into different states using 3 stages of work steps (cf. Fig. 7):

Stage I:

In this stage, the traffic flow in the major stream is divided into 2 states which excludes each other:

- **Queuing** and **Queuing-free**
  In the state of Queuing, the vehicles in the major stream stay at the stop line or are within discharging operation. Departure from the minor stream is not possible in the state of Queuing (including discharge queuing). In the state of Queuing-free all vehicles in the major stream are in motion. Departure from the minor stream is in the state of Queuing-free possible but dependent on the traffic intensity and bunching situation within the major stream. Denoting the probability for the state of Queuing by
  p_S = Pr(Queuing),
  the probability for the state of Queuing-free is then
  $p_{0,S}$==Pr(Queuing-free) = 1 - p_S.

Stage II:

In the stage II, the traffic flow in the Queuing-free state is in turn divided into 2 sub-states which excludes each other:

- **Bunching** and **Bunching-free** under the condition of Queuing-free
  In the state of Bunching, the vehicles in the major stream is in motion with the minimum gaps τ. Departure from the minor stream is not possible in the state of Bunching. In the state of Bunching-free the gaps between the vehicles are large than τ and distributed by chance. Departure from the minor stream is in the state of Bunching-free possible but dependent on the traffic intensity within the major stream in this state. Denoting the probability for the state of Bunching under the condition of Queuing-free by
  p_B = Pr(Bunching | Queuing-free),
  the probability for the state of Bunching-free the condition of Queuing-free is then
  $p_{0,B}$ = Pr(Bunching-free | Queuing-free) = 1 - p_B.

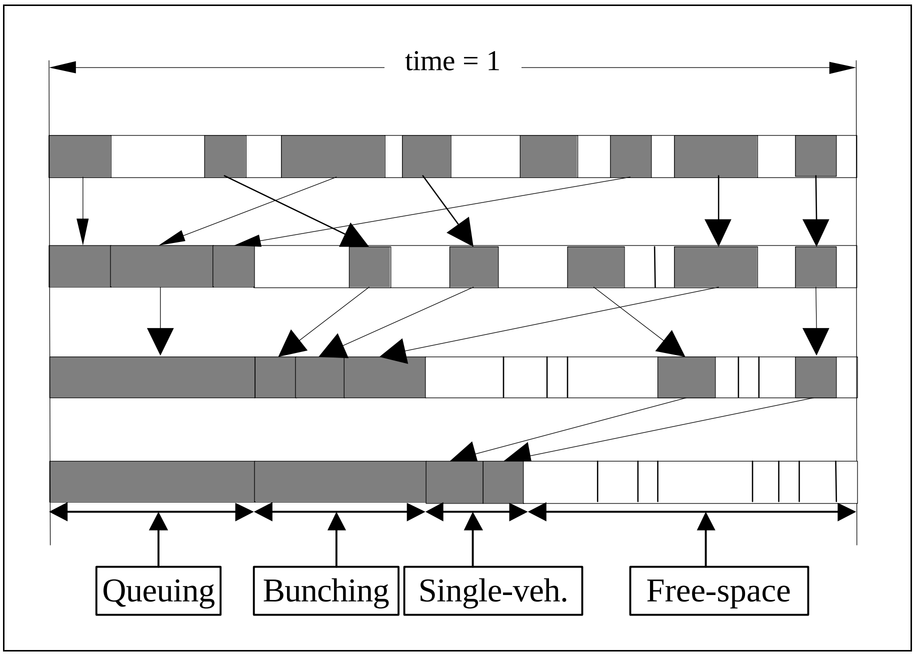

Fig. 7 -
States within the major stream.

For further consideration of the probabilities of the single states, the blocking regimes by Queuing, Bunching, Single-vehicle, and the regime of Free-space are pulled together.

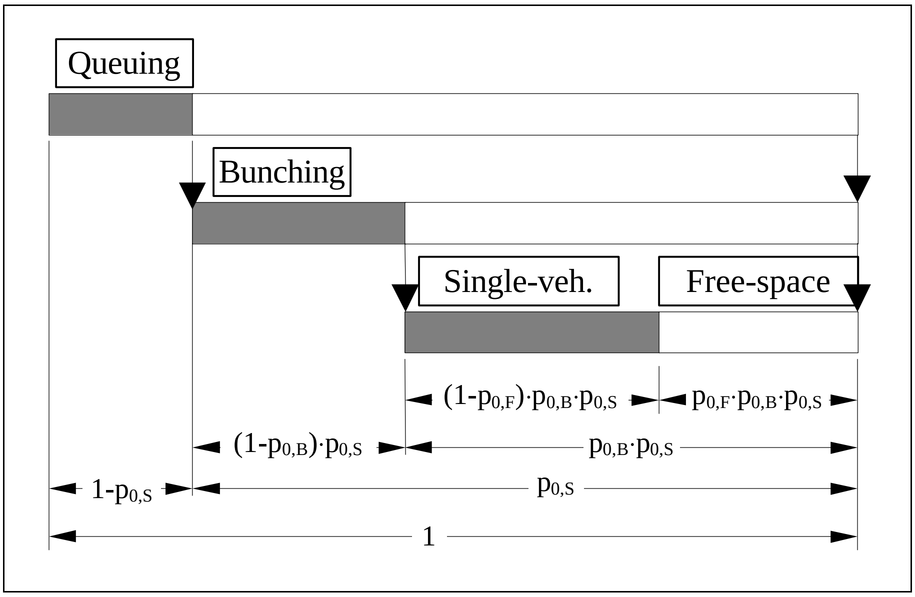

Fig. 8
States in the major stream and their probabilities

Stage III:

In the stage III, the traffic flow in the Bunching-free state under the condition of Queuing-free is divided again into 2 sub-sub-states which excludes each other:

- **Single-vehicle** and **Vehicle-free** (Free-space) under the condition of (Bunching-free | Queuing-free)
  In the state of Single-vehicle, vehicles in the major stream are moving independently from each other. In the front of a vehicle, a time period of the length t_0 is closed for the minor stream. The total closing time by the vehicles in the major stream is the sum of the set $\{t_0\}$. Departure from the minor stream is not possible for the state of Single-vehicle. In the state of Free-space there is no vehicle in the major stream. Departure from the minor stream in the state of Free-space is carried out with the saturation capacity $C_s=1/t_f$.

Denoting the probability for the state of Single-vehicle under the condition of (Bunching-free | Queuing-free) by

p_F = Pr[Single-vehicle | (Bunching-free | Queuing-free)],

the probability for the state of Vehicle-free (Free-space) under the condition of (Bunching-free | Queuing-free) is then

$p_{0,F}$ = Pr[Vehicle-free | (Bunching-free | Queuing-free)] = 1 - p_F.

Thus, the major stream can be divided into four regimes 1) that of state of Free-space (Vehicle-free), 2) that of state of Single-vehicle, 3) that of state of Bunching, and 4) that of state of Queuing. According to the definition of the conditioned probabilities, the probabilities $p_{0,S}$, $p_{0,B}$ and $p_{0,F}$ are completely independent of each other (cf. Figs. 7 and 8). They are to be determined according to the queuing theory.

Accordingly, the formula for the determination of capacity of the minor stream reads

C = (saturation capacity | no hindrance)
× Pr(no hindrance by Queuing)
× Pr(no hindrance by Bunching | no hindrance by Queuing)
× Pr[no hindrance by Single -vehicle | (no hindrance by Bunching | no hindrance by Queuing)]
= Pr[Vehicle-free | (Bunching-free | Queuing-free)]

$$= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \qquad (38)$$

with C_s = saturation capacity | no hindrance = capacity in the Free-space state

<u>3.1.1 Probability of the Queuing-free state, $p_{0,S}$.</u> The probability for the state of Queuing-free in the major stream p_S can in general (approximately according to the M/G/1 queuing system) be estimated with the saturation degree x_p. The probability for the Queuing-free state in the major stream $p_{0,S}$ then reads

$$p_{0,S} = 1 - p_S = 1 - x_p \qquad (39)$$

<u>3.1.2 Probability of the Bunching-free state under the condition of Queuing-free, $p_{0,B}$.</u> One can assume that bunching formation in the traffic in motion within the major stream is independent of the queuing saturation. Accordingly, it is true

Pr(Bunching-free | Queuing-free) = Pr(Bunching-free)

The probability of Bunching p_B in the major stream is simple the portion of the sum of the minimum gap τ for all vehicles. Thus

$$p_B = \sum_{i=1}^{q_p} \tau_i = q_p \cdot \bar{\tau}$$

The probability for Bunching-free state within the major stream $p_{0,B}$ reads then

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \qquad (40)$$

<u>3.1.3 Probability of the Vehicle-free (Free-space) state under the condition of (Bunching-free | Queuing-free), $p_{0,F}$.</u> The probability for the state of Free-space under the condition of (Bunching-free | Queuing-free), $p_{0,F}$ (the probability that no hindrance by single vehicles occurs), is only dependent on the traffic intensity q_f. It is identical to the probability that the gap in the

major stream t is larger than zero-gap t_0 under the condition that the gap t is larger than the minimum gap τ (cf., Fig.4). That is:

$$p_{0,F} = \Pr(t > t_0 \mid t > \tau) = \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \tag{41}$$

3.1.4 Capacity in the Vehicle-free (Free-space) state C_s. The capacity for the minor stream is in the Vehicle-free state the reciprocal of the mean service time of the queuing system. The mean service time in the vehicle free state is equal to the mean move-up time t_f. That is:

$$C_s = \frac{1}{\bar{t}_f} \tag{42}$$

3.1.5 Capacity C. The general formula for the determination of capacity now reads

$$C = C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} = C_s \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \tag{43}$$

With shifted-exponentially distributed gaps t in the major stream it is valid for systems with continuous departure and fixed t_g, t_f (and consequently t_0 is also fixed) and τ

$$C_s = \frac{1}{t_f} \tag{44}$$

and

$$\frac{\Pr(t > t_0)}{\Pr(t > \tau)} = e^{-q_f \cdot (t_0 - \tau)} \tag{45}$$

The resulted formula for the determination of capacity reads

$$C = C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} = (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{e^{-q_f \cdot (t_0 - \tau)}}{\bar{t}_f} \tag{46}$$

3.1.6 Generalization. In general it is true

$$C = C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \tag{47}$$

For systems with continuous departure, one has

$$C_s = \frac{1}{\bar{t}_f} \tag{48}$$

$$p_{0,S} = 1 - x_p \tag{49}$$

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \tag{50}$$

and

$$p_{0,F} = \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \tag{51}$$

with $\bar{t}_0 \approx \bar{t}_g - \left(\frac{\bar{t}_f}{2} + \frac{\sigma^2_{t_f}}{2 \cdot \bar{t}_f} \right)$ (cf. also Heidemann and Wegmann, 1997)

That is:

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= \frac{1}{\bar{t}_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{\Pr(t > t_0)}{\Pr(t > \tau)} \\ &= \frac{1}{\bar{t}_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{1 - F(t = t_0)}{1 - F(t = \tau)} \end{aligned} \tag{52}$$

with F(t) = Distribution function of the gaps t within major stream

For instance, for the shifted-hyper-Erlang-distributed gaps t within the major stream with fixed t_f, t_0 and τ, the capacity reads

$$\begin{aligned} C &= \frac{1}{t_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \frac{1 - F(t = t_0)}{1 - F(t = \tau)} \\ &= \frac{1}{t_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \frac{A \cdot e^{-q_{f,1} \cdot (t_0 - \tau)} + (1 - A) \cdot e^{-k \cdot q_{f,2} \cdot (t_0 - \tau)} \cdot \sum_{i=0}^{k-1} \frac{(k \cdot q_{f,2} \cdot (t_0 - \tau))^i}{i!}}{1 - 0} \\ &= \frac{1}{t_f} \cdot (1 - x_p) \cdot (1 - q_p \cdot \tau) \cdot \left(A \cdot e^{-q_{f,1} \cdot (t_0 - \tau)} + (1 - A) \cdot e^{-k \cdot q_{f,2} \cdot (t_0 - \tau)} \cdot \sum_{i=0}^{k-1} \frac{(k \cdot q_{f,2} \cdot (t_0 - \tau))^i}{i!} \right) \end{aligned} \tag{53}$$

with A = Portion of the negative-exponentially distributed gaps
$q_{f,1}$ = Traffic intensity in the portion of traffic with negative-exponentially distributed gaps
$q_{f,2}$ = Traffic intensity in the portion of traffic with Erlang-distributed gaps

Here, the relationship

$$\frac{1}{q_f} = A \cdot \frac{1}{q_{f,1}} + (1 - A) \cdot \frac{1}{q_{f,2}}$$

is true.

If the gaps t in Bunching-free state are exponentially distributed, then the following is true

$$p_{0,F} = L(t_0(q_f)) \cdot L(\tau(-q_f)) \tag{54}$$

for inconsistent departure behavior and

$$p_{0,F} = \frac{1}{L(t_0(-q_f)) \cdot L(\tau(q_f))} \tag{55}$$

for consistent departure behavior.

3.2 System with discrete departure

Analogously for systems with discrete departure (cf. eq.(52))

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= C_s \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{\Pr(t > t_g)}{\Pr(t > \tau)} \\ &= C_s \cdot (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{1 - F(t = t_g)}{1 - F(t = \tau)} \end{aligned} \quad (56)$$

is valid.

Normally, the capacity in Free-space state C_s for systems with discrete departure cannot be expressed explicitly. With the assumption that the gaps t in the Bunching-free state are exponentially distributed one receives

$$C_s = \frac{q_f}{1 - L\left(t_f(q_f)\right)} \quad (57)$$

$$p_{0,S} = 1 - x_p \quad (58)$$

$$p_{0,B} = (1 - q_p \cdot \bar{\tau}) \quad (59)$$

$$p_{0,F} = L\left(t_g(q_f)\right) \cdot L\left(\tau(-q_f)\right) \quad (60)$$

for inconsistent departure behavior and

$$p_{0,F} = \frac{1}{L\left(t_g(-q_f)\right) \cdot L\left(\tau(q_f)\right)} \quad (61)$$

for consistent departure behavior.

For fixed t_g, t_f and τ it is valid for discrete departure

$$C_s = \frac{q_f}{1 - e^{-q_f \cdot t_f}} \quad (62)$$

and

$$p_{0,F} = e^{-q_f \cdot \left(t_g - \tau\right)} \quad (63)$$

The resulted formula for the determination of capacity reads

$$\begin{aligned} C &= C_s \cdot p_{0,S} \cdot p_{0,B} \cdot p_{0,F} \\ &= (1 - x_p) \cdot (1 - q_p \cdot \bar{\tau}) \cdot \frac{q_f \cdot e^{-q_f \cdot (t_g - \tau)}}{1 - e^{-q_f \cdot t_f}} \end{aligned} \quad (64)$$

4 MAJOR STREAMS CONTAINING MORE THAN ONE TRAFFIC LANES

According to the derivation in the section 3, the traffic flow within the major stream can be divided into four states for which the probabilities can be calculated separately. From these probabilities, the portion of the states in which no hindrance in the major stream occurs to the minor stream can be obtained. The capacity for the minor stream can be calculated from the multiplication of the saturation capacity C_s and the portion of the state without hindrance.

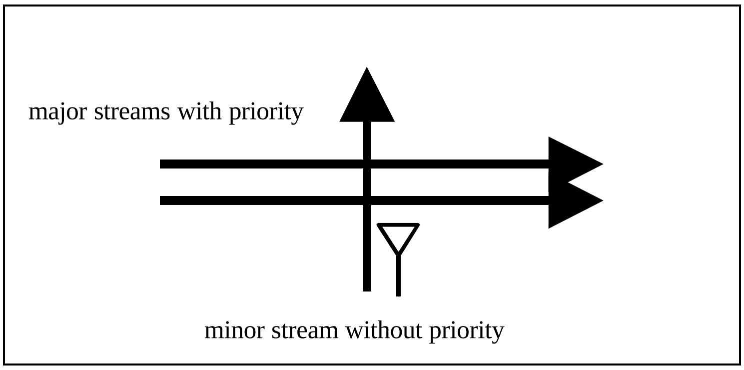

Fig. 9 - System with two major streams with parallel configuration

According to the same principle, the capacity for systems with a major stream containing more than one traffic lanes can also be determined.

The traffic states in major traffic lanes with parallel configuration are completely independent of each other. Therefore, it is valid for systems with n major traffic lanes (cf. Fig.**9**):

$$p_{0,S}^* = \prod_{i=1}^{n} p_{0,S,i} \quad , \quad p_{0,B}^* = \prod_{i=1}^{n} p_{0,B,i} \quad \text{and} \quad p_{0,F}^* = \prod_{i=1}^{n} p_{0,F,i} \qquad (65)$$

and accordingly,

$$\begin{aligned} C &= C_s \cdot p_{0,S}^* \cdot p_{0,B}^* \cdot p_{0,F}^* \\ &= C_s \cdot \prod_{i=1}^{n} p_{0,S,i} \cdot \prod_{i=1}^{n} p_{0,B,i} \cdot \prod_{i=1}^{n} p_{0,F,i} \end{aligned} \qquad (66)$$

with

$$C_s = \frac{1}{\bar{t}_f} \qquad (67)$$

for continuous departure and

$$C_s = \frac{q_f^*}{1 - L\left(t_f(q_f^*)\right)} = C_s(q_f^*) \qquad (68)$$

$$q_f^* = \sum_{i=1}^{n} q_{f,i}$$

for discrete departure.

In the equations, $p_{0,X,i}$ is the probability that the state X in stream i does not occur.

For fixed t_f, eq.(68) turns to

$$C_s = \frac{q_f^*}{1 - e^{-q_f^* \cdot t_f}} \tag{69}$$

5 APPLICATION OF THE PROCEDURE

5.1 Example for of multi-lane major roads

The traffic lanes on a major road can be considered as a parallel configuration. According to eqs. (66) to (69) one obtains C for a major road containing n traffic lanes with fixed t_g, t_f and τ

$$\begin{aligned} C &= \prod_{i=1}^{n} \left(1 - \tau \cdot q_{p,i}\right) \cdot \frac{1}{t_f} \cdot \prod_{i=1}^{n} \exp\left(-q_{f,i} \cdot (t_{0,i} - \tau_i)\right) \\ &= \prod_{i=1}^{n} \left(1 - \tau \cdot q_{p,i}\right) \cdot \frac{1}{t_f} \cdot \exp\left(-\sum_{i=1}^{n} \left(q_{f,i} \cdot (t_{0,i} - \tau_i)\right)\right) \end{aligned} \tag{70}$$

with $t_{0,i} = t_{g,i} - \frac{t_f}{2}$

for continuous departure or

$$\begin{aligned} C &= \prod_{i=1}^{n} \left(1 - \tau \cdot q_{p,i}\right) \cdot \prod_{i=1}^{n} \exp\left(-q_{f,i} \cdot (t_{g,i} - \tau_i)\right) \cdot \frac{\sum_{i=1}^{n} \left(q_{f,i}\right)}{1 - \exp\left(-\sum_{i=1}^{n} \left(q_{f,i}\right) \cdot t_f\right)} \\ &= \prod_{i=1}^{n} \left(1 - \tau \cdot q_{p,i}\right) \cdot \frac{\sum_{i=1}^{n} \left(q_{f,i}\right) \cdot \exp\left(-\sum_{i=1}^{n} \left(q_{f,i} \cdot (t_{g,i} - \tau_i)\right)\right)}{1 - \exp\left(-\sum_{i=1}^{n} \left(q_{f,i}\right) \cdot t_f\right)} \end{aligned} \tag{71}$$

for discrete continuous departure.

5.2 Example for roundabouts

A approach with n_e traffic lanes to a roundabout with n_c circulation lanes is considered.

Setting $q_{p,i}=q_c/n_c$, $t_{g,i}=t_g$, $\tau_i=\tau$, and $\varphi_i = 1 - \tau_i \cdot q_{p,i}$ for all major streams, one obtains for the approach at roundabouts the universal capacity formula (cf. Wu, 1997b)

$$C = n_e \cdot \left(1 - \frac{\tau \cdot q_c}{n_c}\right)^{n_c} \cdot \frac{1}{t_f} \cdot \exp\left(-q_c \cdot (t_0 - \tau)\right) \tag{72}$$

with C = total capacity of the approach
q_c = total traffic intensity in the circulation lanes
t_0 = t_g - t_f/2

The values t_g=4.12s, t_f=2.88s,and τ=2.10s are obtained at roundabouts in Germany

6 SUMMARIES

A new procedure for the determination of capacity is presented here. This new procedure can be applied for the conditions

- arbitrary many major streams with different critical gaps t_g and minimum gaps τ
- arbitrary distribution of the gaps t in the major streams
- arbitrary distribution of critical gaps t_g, move-up times t_f and minimum gaps τ if the corresponding Laplace transform of the distribution are given
- arbitrary queuing and bunching saturations in the major streams

The new procedure provides a generalized form of all the usual formulae for calculating the capacity at unsignalized intersections. The usual formula are reproduced by the new procedure if the corresponding parameters are set. For practical uses, the procedure should be calibrated and validated with measurements or simulations. The initial calibrations and validation for roundabouts (Wu, 1997b) already shows potential of the new procedure.

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