Argument Evaluation Based on Proportionality

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Argument Strength
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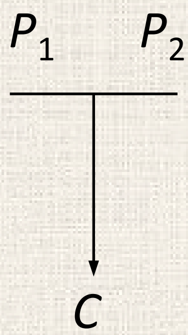
The strength of structured arguments

I. Syntax
II. Evaluation
III. Attack relation

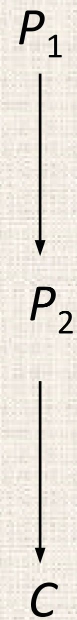
I. Syntax

Classical diagrams of arguments

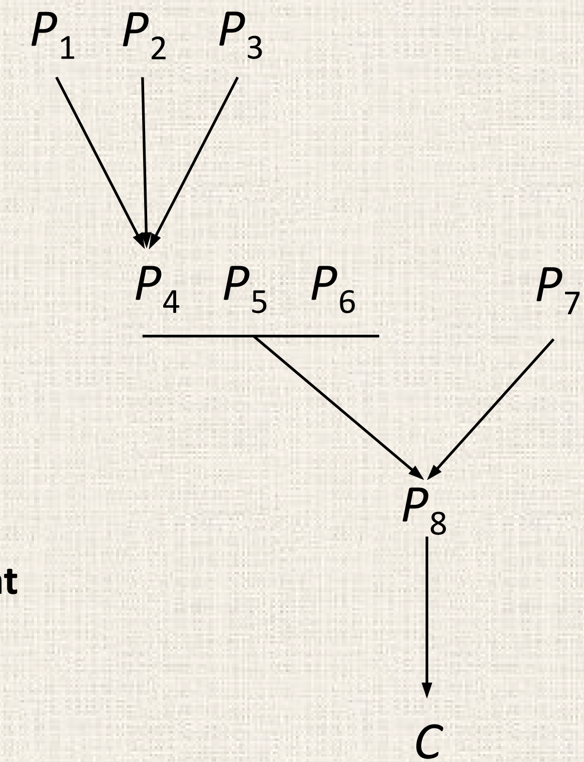
Simple argument



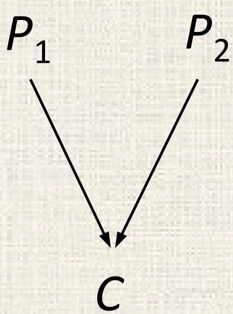
Linked argument



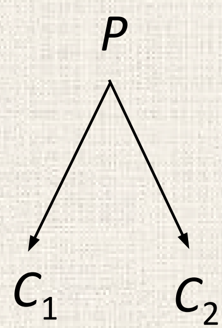
Serial argument



Multilevel complex argument



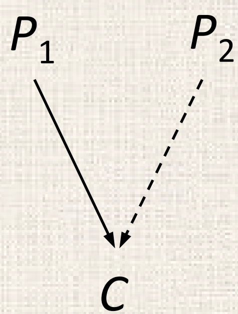
Convergent argument



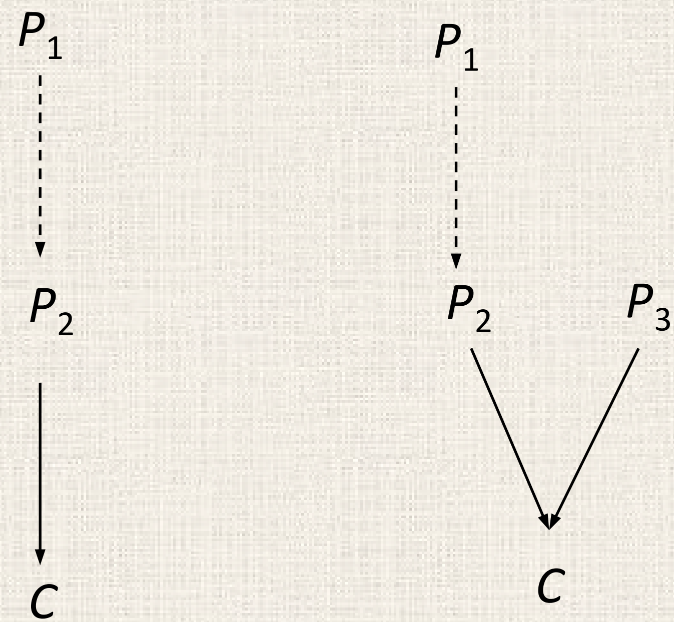
Divergent argument

Conductive arguments (i.e. ***pro-contra***, *cf*. Walton & Gordon 2015)

Con-argument

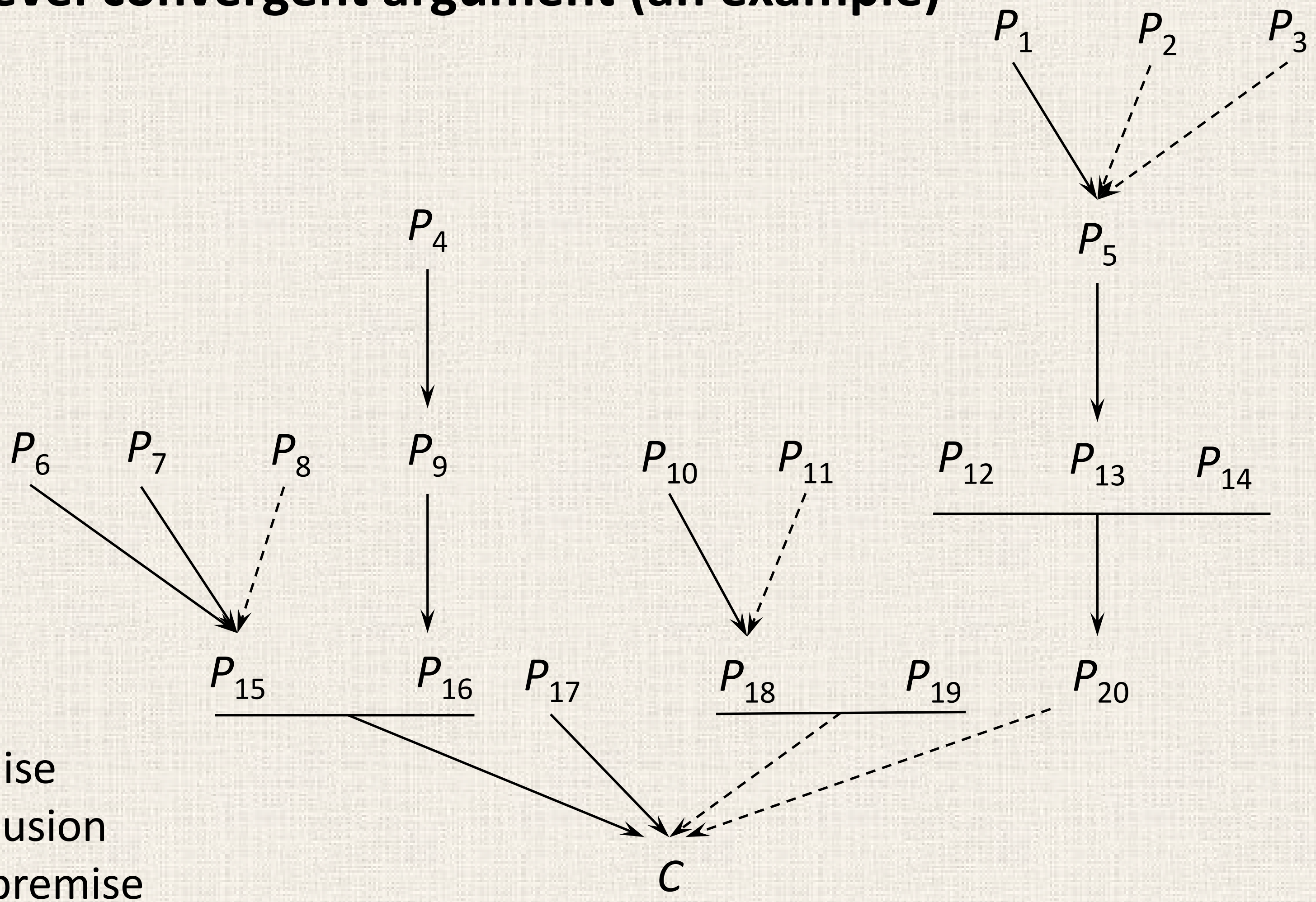


Conductive argument



problematic cases

Multilevel convergent argument (an example)



- premise
- conclusion
- first premise
- intermediate conclusion
- final conclusion, final argument
- <u>atomic argument</u> (= either simple or linked)

Formal structure of arguments

Two kinds of inference:

- *pro*-premises *support* conclusions;
- *con*-premises *deny* (*contradict*) conclusions.

Formal representation (Selinger 2014, 2015)**:**

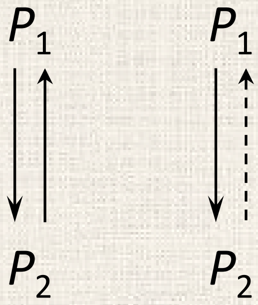
- Let L be a *language*, i.e. a set of sentences.

➢ *Sequents* are any tuples of the form <P, c, d>, where:

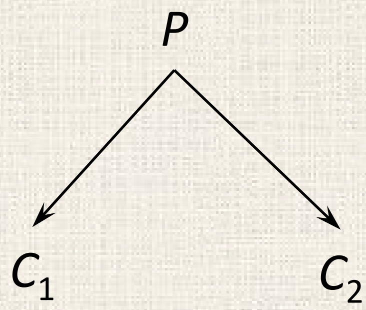
- P is a finite, non-empty set of sentences of L (*premises*) ;
- c is a single sentence of L (*conclusion*);
- d is a Boolean value (1 in *pro*-sequents and 0 in *con*-sequents).

➢ *Argumentation structures* (*arguments*) are any finite and non-empty sets of sequents.

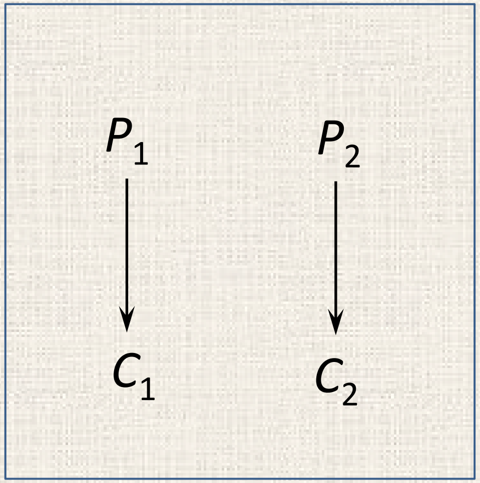
Atypical argumentation structures:



circularity



divergence



incoherence

- arguments can have less or more than one final conclusion
- in what follows arguments will be assumed to be coherent, non-divergent and non-circular

II. Numerical evaluation

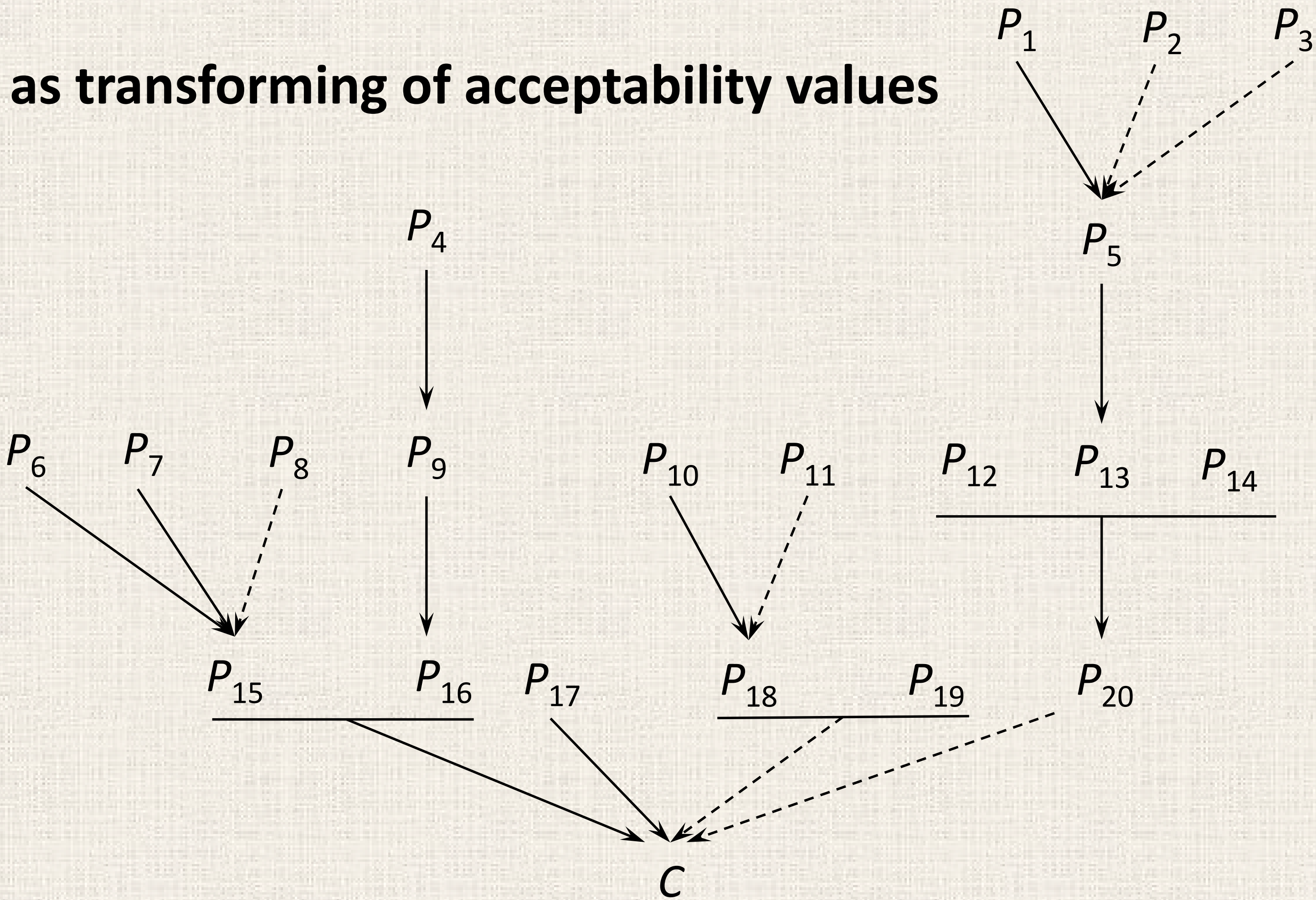
Evaluation. Formal preliminaries

- We assume that *L* contains the negation and the conjunction connectives;
- v: $L' \to [0, 1]$, where $L' \subseteq L$, is *evaluation function* — $v(\alpha)$ is (*the degree of*) *acceptability of* α;
- w: $L \times L \to [0, 1]$ is *conditional acceptability* — $w(\alpha/\beta)$ is the acceptability of α under the condition that $v(\beta) = 1$;

Question: should *w* be a partial function?

- $v(\neg\alpha) = 1 - v(\alpha)$ — *postulate of rationality*;
- If some premises deny α then the evaluation of α is based on the evaluation of $\neg\alpha$ in the corresponding sequent, in which these premises support $\neg\alpha$.

Evaluation as transforming of acceptability values



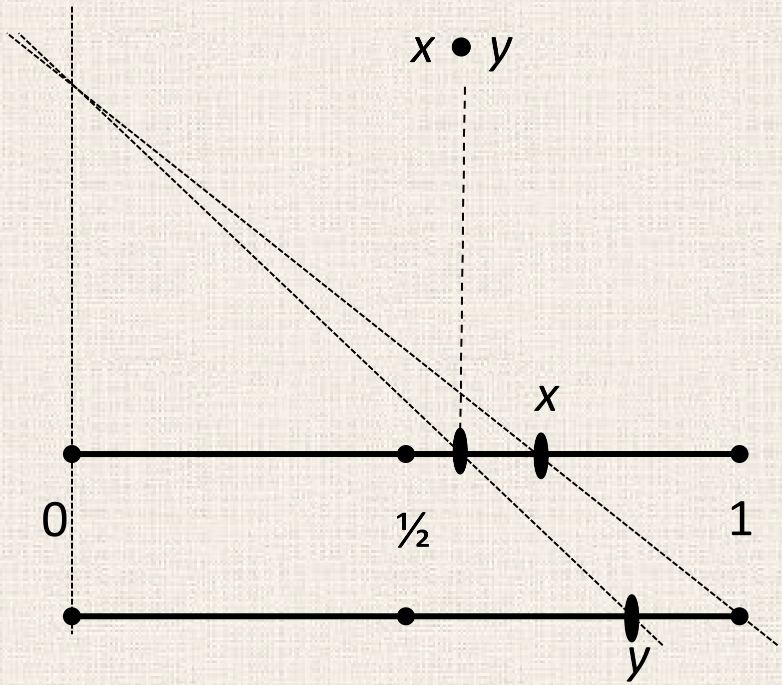
- the evaluation function is defined for the first premises
- the acceptability of the first premises is transformed step by step to the acceptability of the final conclusion
- formally, in each step the domain of the evaluation function is extended to the set containing new conclusion

The principle of proportionality

The strength of argument should vary proportionally to the values assigned to its components.

Evaluation of premises

x, y are the acceptability values of some two premises

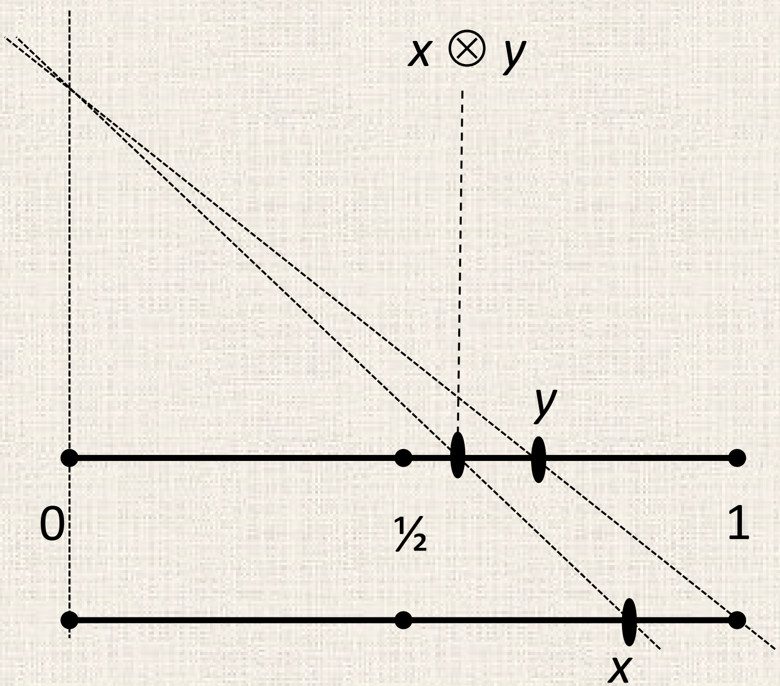


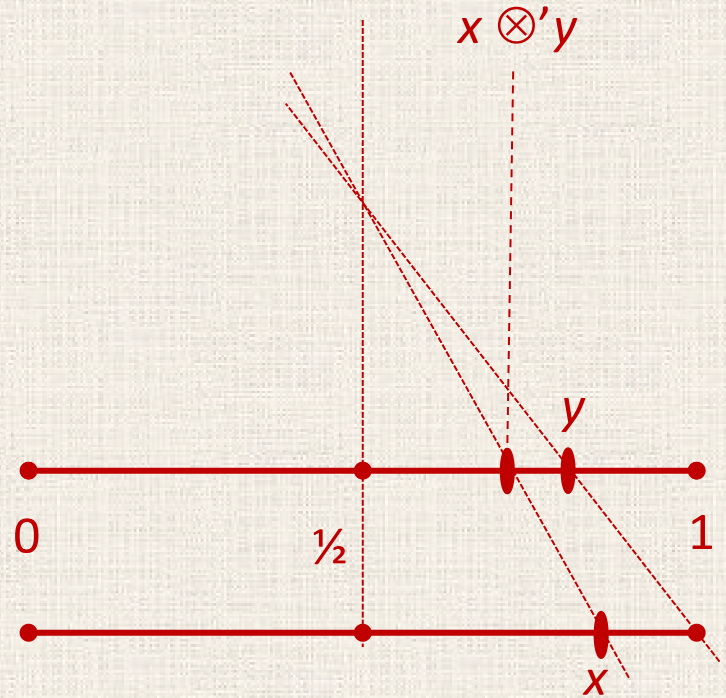
$$\frac{(x \bullet y)}{x} = \frac{y}{1}$$

$x \bullet y = xy$

Evaluation of atomic arguments

x is the acceptability of (the set) of premises;
y is the conditional acceptability (conclusion/premises)
x, y > ½





$$\frac{(x \otimes y)}{y} = \frac{x}{1}$$

$$\frac{(x \otimes' y) - ½}{x - ½} = \frac{y - ½}{½}$$

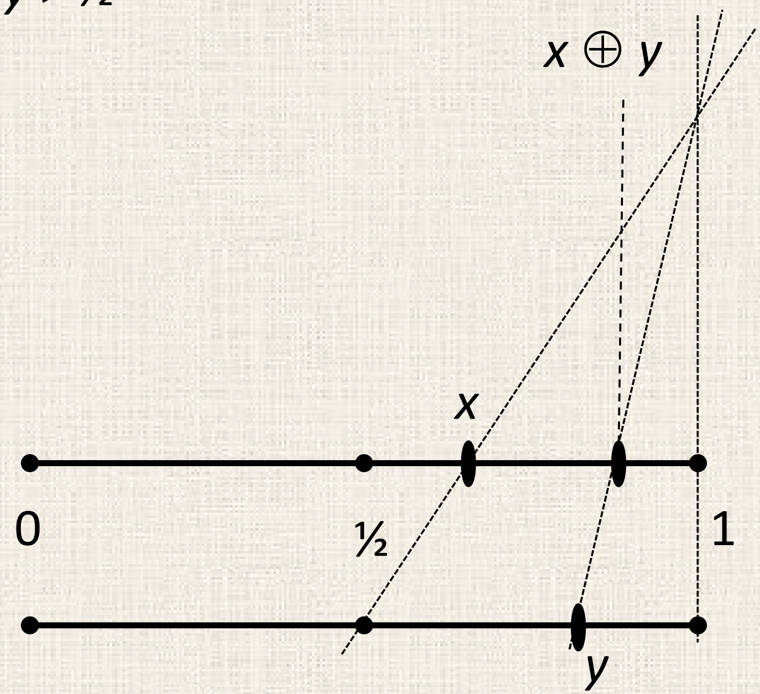
$x \otimes y = xy$

$x \otimes' y = 2xy - x - y + 1$

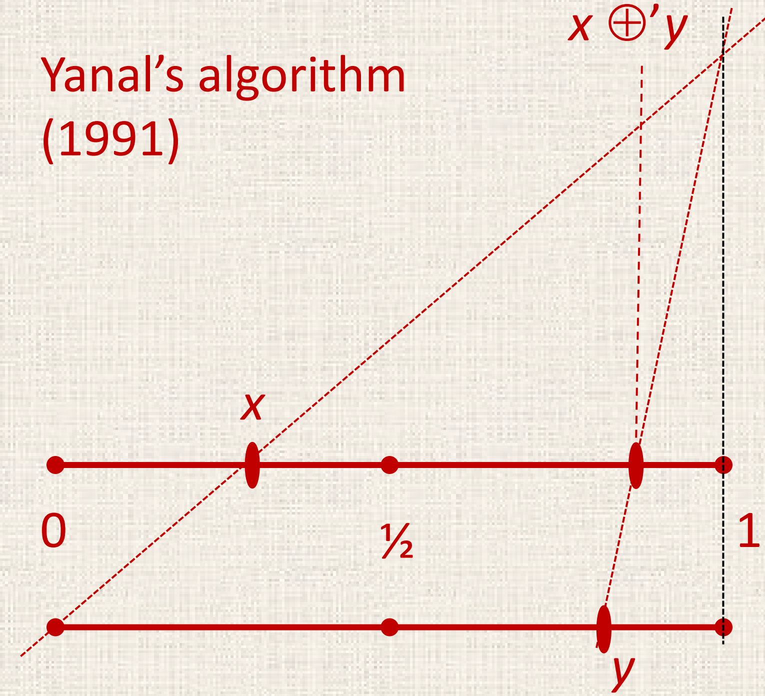
Evaluation of convergent *pro*-arguments

x, y – acceptability values of two converging arguments

$x, y > ½$



Yanal's algorithm (1991)



$$\frac{(x \oplus y) - x}{1 - x} = \frac{y - ½}{½}$$

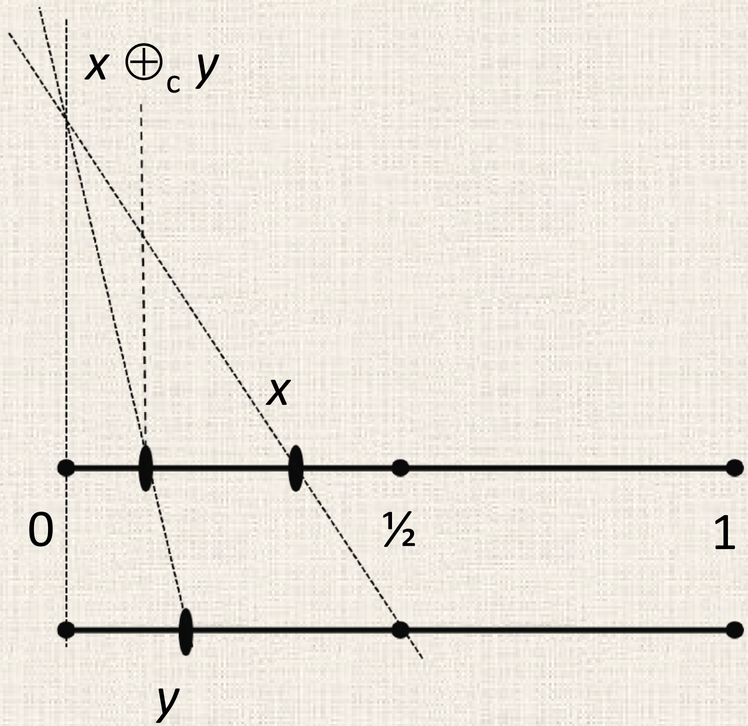
$x \oplus y = 2x + 2y - 2xy - 1$

$$\frac{(x \oplus' y) - x}{1 - x} = \frac{y}{1}$$

$x \oplus' y = x + y - xy$

Evaluation of convergent *con*-arguments

$x, y < ½$



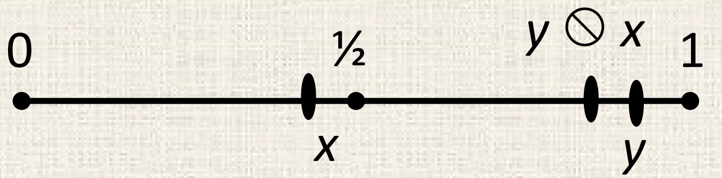
$$\frac{(x \oplus_c y)}{x} = \frac{y}{½}$$

$x \oplus_c y = 2xy$

$x \oplus_c y = 1-[(1-x) \oplus (1-y)] = 2xy$

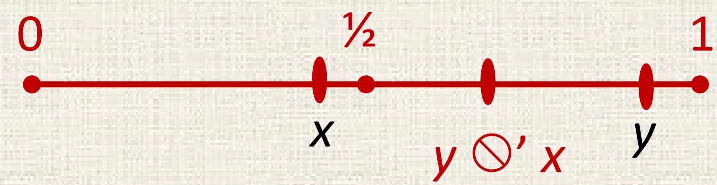
Evaluation of conductive arguments

$x < ½$ is the acceptability of all convergent *con*-arguments
$y > ½$ is the acceptability of all convergent *pro*-arguments



$y \obslash x = (y - ½) - (½ - x) + ½$
$y \obslash x = y + x - ½$

$y \obslash' x$ as the arithmetic mean of x and y



$y - (y \obslash' x) = (y \obslash' x) - x$
$y \obslash' x = ½\,(y + x)$

Evaluation of atomic arguments

Let $\wedge A$ be the conjunction of all the propositions belonging to a finite set A.

Let $\mathbf{A} = \{<P, c, d>\}$, where $P \subseteq dom(v)$, $c \notin dom(v)$, and d is a Boolean value.

The function $v_{\mathbf{A}}$ is the following extension of v to the set $dom(v) \cup \{c\}$:

- If $d = 1$ then $v_{\mathbf{A}}(c) = v(\wedge P) \cdot w(c/\wedge P)$;
- If $d = 0$ then $v_{\mathbf{A}}(c) = 1 - v(\wedge P) \cdot w(\neg c/\wedge P)$.

The value $v_{\mathbf{A}}(c)$ can be called *the (logical) strength or force of the argument* **A**.

Note: the strength of $\{<P, c, 0>\} = 1 -$ the strength of $\{<A, \neg c, 1>\}$.

Acceptability of arguments:

- If **A** is a *pro*-argument then it is acceptable iff $v_{\mathbf{A}}(c) > ½$;
- If **A** is a *con*-argument then it is acceptable iff $v_{\mathbf{A}}(c) < ½$.

Evaluation of convergent arguments

Let **A** be an argument, α its final conclusion, and let $\mathbf{A} = \mathbf{A_1} \cup \mathbf{A_2}$, where both $\mathbf{A_1}$ and $\mathbf{A_2}$ have the same final conclusion *c*, they are coherent and acceptable, and all their sequents whose conclusion is *c* are only either *pro*- or *con*-sequents.

- If both $\mathbf{A_1}$ and $\mathbf{A_2}$ are *pro*, and $v_{\mathbf{A1}}(c), v_{\mathbf{A2}}(c) > ½$, then $v_{\mathbf{A}}(c) = v_{\mathbf{A1}}(c) \oplus v_{\mathbf{A2}}(c)$
- If both $\mathbf{A_1}$ and $\mathbf{A_2}$ are *con*, and $v_{\mathbf{A1}}(c), v_{\mathbf{A2}}(c) < ½$, then $v_{\mathbf{A}}(c) = v_{\mathbf{A1}}(c) \oplus_c v_{\mathbf{A2}}(c) =$
  $= 1 - (1 - v_{\mathbf{A1}}(c)) \oplus (1 - v_{\mathbf{A2}}(c))$

where

$$x \oplus y = 2x + 2y - 2xy - 1$$
$$x \oplus_c y = 2xy.$$

Note: The operations $\oplus$ and $\oplus_c$ are both commutative and associative, therefore the strengths of any number of convergent arguments can be added in any order.

Evaluation of conductive arguments

Let **A** be an argument, α its final conclusion, and let $\mathbf{A} = \mathbf{A}_{\mathbf{pro}} \cup \mathbf{A}_{\mathbf{con}}$, where all the sequents of $\mathbf{A}_{\mathbf{pro}}$ whose conclusion is *c* are only *pro*-sequents and all the sequents of $\mathbf{A}_{\mathbf{con}}$ whose conclusion is *c* are only *con*-sequents.

We assume that both $\mathbf{A}_{\mathbf{pro}}$ and $\mathbf{A}_{\mathbf{con}}$ are coherent and acceptable, i.e. $v_{\mathbf{Apro}}(c) > ½$ and $v_{\mathbf{Acon}}(c) < ½$.

- If $v_{\mathbf{Apro}}(c) < 1$, and $v_{\mathbf{Acon}} > 0$, then $v_{\mathbf{A}}(c) = v_{\mathbf{Apro}}(c) \oslash v_{\mathbf{Acon}}(c)$

where $y \oslash x = y + x - ½$;

- If $v_{\mathbf{Apro}}(c) = 1$, and $v_{\mathbf{Acon}}(c) \neq 0$, then $v_{\mathbf{A}}(c) = 1$;
- If $v_{\mathbf{Apro}}(c) \neq 1$, and $v_{\mathbf{Acon}}(c) = 0$, then $v_{\mathbf{A}}(c) = 0$;
- If $v_{\mathbf{Apro}}(c) = 1$, and $v_{\mathbf{Acon}}(c) = 0$, then $v_{\mathbf{A}}(c)$ is not computable.

III. Attack relation (elementary cases)

Attack relation between arguments. Rebuttals, underminers, undercutters (Prakken 2010)

- Attack on argument **conclusion**:

  $\{<P_1, c, \mathrm{d}>\}$ can attack (*rebut*) $\{<P_2, c, 1 - d>\}$

  $\{<P_1, c, \mathrm{d}>\}$ can attack (*rebut*) $\{<P_2, c', d>\}$, where $c' = \neg c$ or $\neg c' = c$

- Attack on argument **premises**:

  $\{<P_1, c_1, 0>\}$ can attack (*undermine*) $\{<P_2, c_2, d>\}$ if $c_1 \in P_2$

  $\{<P_1, c_1, 1>\}$ can attack (*undermine*) $\{<P_2, c_2, d>\}$ if $c_1' \in P_2$, where $c_1' = \neg c_1$ or $\neg c_1' = c_1$

- Attack on the **relationship between argument premises and argument conclusion**:

  undercutting defeaters

Successful attack on conclusion. Rebuttals

An argument **A** *rebuts (the conclusion of)* an argument **B** iff

- **A** = $\{<P_1, c, d>\}$,
- **B** = $\{<P_2, c, 1 - d>\}$
- either $d = 0$ and $1 - v_{\mathbf{A}}(c) \geq v_{\mathbf{B}}(c)$, or $d = 1$ and $1 - v_{\mathbf{A}}(c) \leq v_{\mathbf{B}}(c)$

or

- **A** = $\{<P_1, c, d>\}$,
- **B** = $\{<P_2, c', d>\}$, where ($c' = \neg c$ or $c = \neg c'$)
- either $d = 0$ and $v_{\mathbf{A}}(c) \leq v_{\mathbf{B}}(c')$, or $d = 1$ and $v_{\mathbf{A}}(c) \geq v_{\mathbf{B}}(c')$.

Note: in the borderline cases, i.e. if the above values are equal, the conclusion is not rebutted, but it is merely questioned.

Successful attack on premises. Underminers

An argument **A** *undermines (a premise of)* an argument **B** iff

- **A** = $\{<P_1, c_1, 0>\}$,
- **B** = $\{<P_2, c_2, d>\}$, where $c_1 \in P_2$ is the attacked premise,
- either $d = 1$ and $v'_{\mathbf{B}}(c_2) = [v(c_1) \oslash v'_{\mathbf{A}}(c_1)] \cdot v(\wedge P_2 - c_1\}) \cdot w(c_2/\wedge P_2) \leq ½$, or $d = 0$ and $v'_{\mathbf{B}}(c_2) = [v(c_1) \oslash v'_{\mathbf{A}}(c_1)] \cdot v(\wedge P_2 - \{c_1\}) \cdot w(\neg c_2/\wedge P_2) \leq ½$, where v' is the function obtained from v by deleting c_1 from its domain;

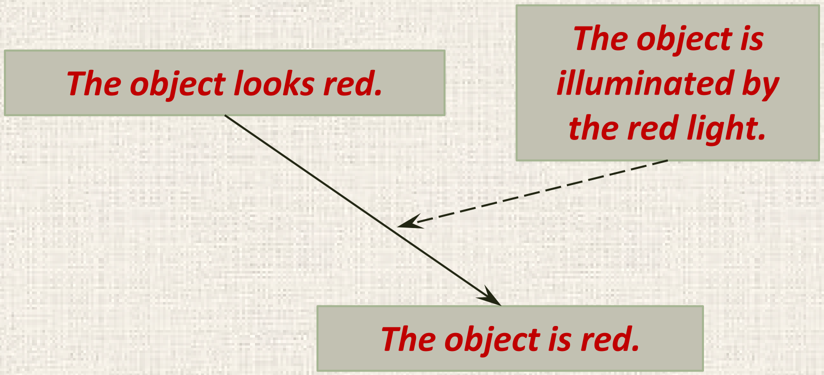
or

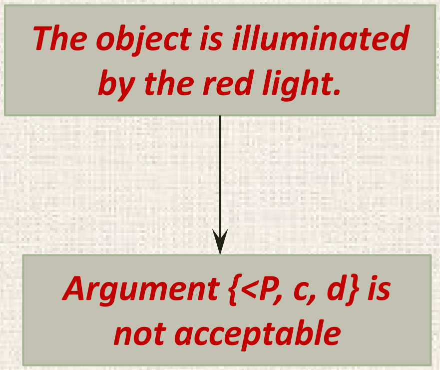
- **A** = $\{<P_1, c_1, 1>\}$,
- **B** = $\{<P_2, c_2, d>\}$, where $c_1' \in P_2$ is attacked ($c_1' = \neg c_1$ or $c_1 = \neg c_1'$),
- either $d = 1$ and $v'_{\mathbf{B}}(c_2) = [v(c_1) \oslash (1 - v'_{\mathbf{A}}(c_1))] \cdot v(\wedge P_2 - \{c_1\}) \cdot w(c_2/\wedge P_2) \leq ½$, or $d = 0$ and $v'_{\mathbf{B}}(c_2) = [v(c_1) \oslash (1 - v'_{\mathbf{A}}(c_1))] \cdot v(\wedge P_2 - \{c_1\}) \cdot w(\neg c_2/\wedge P_2) \leq ½$, where v' is the function obtained from v by deleting c_1 and c_1' from its domain.

Undercutters. Formal representation

Pollock's example of undercutting defeater (1987) :

The object looks red, thus it is red, unless it is illuminated by a red light.





1) $\langle P, c, d, R\rangle$, where R is a set of (linked) rebuttals.

If R is non-empty then:

$\{\langle P, c, d, R\rangle\}$ can undercut $\{\langle P, c, d, \varnothing\rangle\}$

2a) $\{\langle R, \{\langle P, c, d\rangle\}$ *is not acceptable*$, 1\rangle\}$

2b) $\{\langle R, \{\langle P, c, d\rangle\}$ *is acceptable*$, 0\rangle\}$

2a) or 2b) can undercut $\{\langle P, c, d\rangle\}$

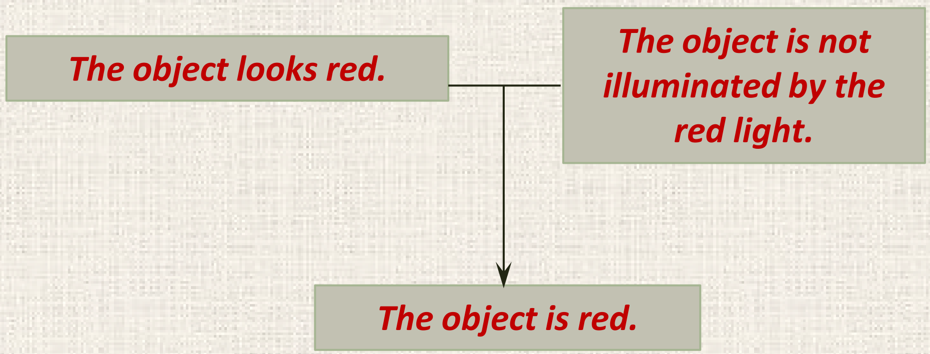
Undercutters. Formal representation

Changing the categorial classification of the attack relation, i.e. including (sets of) sentences to its domain:

R can attack (undercut) {<*P*, *c*, *d*>}

The sentence '*the object X is illuminated by a red light*' can undercut the argument '*the object X looks red, thus it is red*'.

Relevance of undercutters. Hybrid arguments (Vorobej 1995)

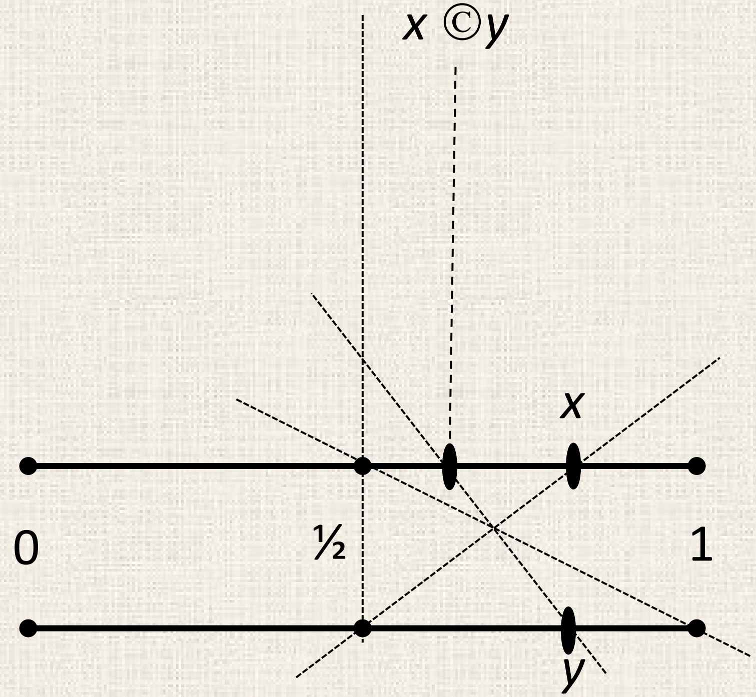


The relevance condition for undercutters:

$w(c/\wedge P \wedge \neg \wedge R) > w(c/\wedge P)$

Undercutters. Evaluation

x is the conditional acceptability of an attacked argument (conclusion/premises);
y is the acceptability of its undercutter;
$x, y > ½$



$$\frac{(x © y) - ½}{x - ½} = \frac{1 - y}{½}$$

$$x © y = 2x + y - 2xy - ½$$

Successful attack on the relationship between premises and conclusion. Undercutters

A set of sentences *R undercuts* an argument **B** iff

- **B** = {<*P*, *c*, 1>},
- *R is a relevant undercutter for* **B**, i.e. $w(c/\wedge P \wedge \neg \wedge R) > w(c/\wedge P)$,
- $v_{\mathbf{B},\, R}(c) = v(\wedge P) \cdot [v(c/\wedge P) \copyright v(\wedge R)] \leq ½$.

or

- **B** = {<*P*, *c*, 0>},
- *R is a relevant undercutter for* **B**, i.e. $w(\neg c/\wedge P \wedge \neg \wedge R) > w(\neg c/\wedge P)$,
- $v_{\mathbf{B},\, R}(c) = v(\wedge P) \cdot [v(\neg c/\wedge P) \copyright v(\wedge R)] \leq ½$.

Note: an unsuccessful undercutting attack can result in strengthening of the attacked argument if its attacker happens to be non-acceptable and the corresponding hybrid argument is stronger than the attacked one.

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