Standard Deviation of Travel Time in a Freeway Network
– a Mathematical Quantifying Tool for Reliability Analysis

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ABSTRACT

Travel time reliability is a new way of looking at congestion and unpredictable variation of travel time. The standard deviation of travel time is a good indicator for investigating reliability of a network. This paper presents a mathematical model dealing with the standard deviation of the total travel time within a freeway network. In general, the distribution of the travel time of links and the distribution of delays at bottlenecks can be described by different probability distributions. The parameters of those distributions can be calibrated by measurements or simulations studies. However, it is hard to calculate the standard deviation or variance of travel time of a route consisting of several consecutive links or bottlenecks. The presented paper shows that under some assumptions the variance of the total route travel time can be calculated as the sum of the variances of the single links or bottlenecks in case that the travel times and the delays are independent of each other. In reality the independency between the consecutive links or bottlenecks may not be satisfied. In this case the variance of the total travel time can also be estimated given the correlation coefficient between the two consecutive links or bottlenecks. Again, this correlation coefficient can be calibrated by measurements or by simulation studies. Once the variance the travel time is known, the standard deviation is also known. Using the proposed model, the standard deviation of travel time and thus the reliability of a freeway network can be quantitatively estimated given the geometric design of the freeway network and the traffic demand.

Keywords: Freeway Network, Travel Time, Reliability, Standard Deviation

INTRODUCTION

Travel time reliability is one of the key indicators for the performance of transport systems. Travel time reliability is a way of looking at congestion and unpredictable variation of travel time. The increased attention for travel time reliability has inspired many researchers working on this subject (e.g. Brownstone and Small, 2005; Clark and Watling, 2005; Kwon et al., 2011; Passier, 2009; Peer et al., 2012; Sweet and Chen, 2011; van Lint et al., 2008). Travel time reliability significantly influences the choice of routes, departure times, and trip links (Abdel-Aty et al., 1996; Bell and Cassir, 2002; Bogers and van Zuylen, 2004; Li et al., 2009). One minute reduction in the standard deviation of travel time and two minutes reduction in the actual travel time can be considered equivalent (Bates et al., 2001). In the first place, travel time reliability is a perception of travelers. With the increasing attention on travel time reliability, many different definitions of travel time reliability have been proposed (Bell et al., 1999; van Lint et al., 2008). These measures relate to properties of the (day-to-day or within-day) travel time distributions, and particularly to the shape of the distribution. There are many candidate measures having very little correlation amongst themselves (van Lint et al., 2008). Bogers (2009) concludes that the most suitable measure for travel time reliability depends on what kind of effects of reliability has to be evaluated. This inconsistency leads to different assessment criteria used by policy evaluations. This may cause ambiguous evaluations. As a reference, the variance or standard deviation is used in this paper for defining the reliability of travel times. The variance/standard deviation describes unambiguously the day-to-day variation of travel time (see Figure 1).

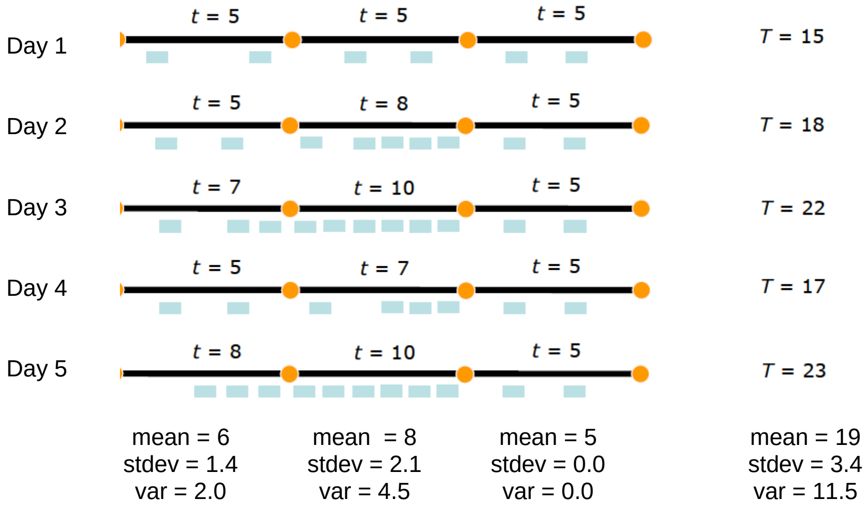

Figure 1. Standard deviation and variance for links and route

The day-to-day variation of travel time can be caused by unexpected severe weather conditions, work zones, and incidents or simply by the stochastic nature of the traffic flow and the variation of capacity (Lorenz and Elefteriadou, 2000; Persaud et al., 1998; Brilon et al., 2005; Elefteriadou et al., 1995). When one or more random events like these lead to traffic congestion, then the standard deviation of travel time and thus the unreliability of the network increases. For a single freeway link with homogeneous characteristics, the free flow travel time and its standard deviation can be easily obtained either by measurements or by existing models. Congestion within a freeway link can be considered as caused by a bottleneck within the link. It is also possible to estimate the distribution of delays occurring at such bottlenecks. In a network consisting of several consecutive components such as freeway links and bottlenecks, the total travel time can be considered as the superposition of free flow travel times of the links and delays of the bottlenecks. Thus, the total travel time of a route is the sum of all free flow travel times of the links and all delays at bottlenecks. However, the standard deviation of the total travel time of a route is not equal to the sum of the standard deviations of travel times or delays within the single links.

This paper presents a mathematical model dealing with the standard deviation of the total travel time within a freeway route. In general, the distribution of free flow travel time of links and the distribution of delays at bottlenecks can be described either by an exponential, a normal, an Erlang, or a Gamma distribution. The parameters of those distributions can be calibrated by measurements or simulation studies. Based on this fact and under some assumptions, the variance of the total travel time of a route can be calculated as the sum of the variances of the single links in case that the travel times and the delays are independent of each other. In reality, the independency between the consecutive links may not exist. In this case the variance of the total travel time of a route can also be estimated if the correlation coefficient between two consecutive links is known. Again, this correlation coefficient can be calibrated by measurements or by simulation studies. Once the variance the travel time is known, the standard deviation is also known.

The proposed model is calibrated and validated using data from the literature. Using the proposed model, the variance or standard deviation of travel time and thus the reliability of a freeway route (also a network) can be quantitatively estimated given the geometric design of the freeway network and the traffic demand.

OVERVIEW OF TRAVEL TIME DISTRIBUTION

Many studies related to fitting travel time distributions from observed travel time data have been conducted (e.g. Al-Deek and Emam, 2006; Herman and Lam, 1974; Polus, 1979; Pu, 2010; Richardson and Taylor, 1978; Susilawati et al., 2012; Wardrop, 1952). Wardrop (1952), for instance, suggested that travel times follow a skewed distribution. Herman and Lam (1974) proposed either the Gamma or lognormal distributions to represent the travel time probability distribution. Richardson and Taylor (1978) found that the observed travel times might be fitted by a lognormal distribution. Polus (1979) concluded that the Gamma distribution was better than normal or lognormal distributions and Al-Deek and Emam (2006) proposed the Weibull distribution to fit observed travel times. Van Lint et al. (2008) depicted travel time distributions with four different shapes based on traffic conditions (free flow, congestion onset, congestion, and congestion dissolve). Pu (2010) concluded that these four shapes of travel time distributions are similar to those of the lognormal distribution and proposed the lognormal distribution. Susilawati et al. (2012) proposed the Burr Type XII distribution for travel time variability on urban roads. Based on the distributions of travel times, a large number of travel time reliability measures have been proposed by previous researchers (e.g. Asakura and Kashiwadani, 1991; Bates et al., 2001; Booz and Hamilton, 1998; Fosgereau and Karlstrom, 2010; Lomax et al., 2003; Pearce, 2001; Tu, 2008; van Lint et al., 2008). If the distribution of travel time is known, the measures of reliability can be defined correspondingly.

Also the Erlang distribution can be used for describing travel time. The Erlang distribution is left skewed and a special case of Gamma distribution. Because of the special property that the sum of single Gamma/Erlang distributions is still Gamma/ Erlang distributed and the total variance is equal to the sum of the variances of the single distributions, the Gamma distribution is used for describing travel time in this paper. In order to account for the lower limit of travel time, a shifted Gamma distribution is used actually. The probability density function (pdf) of the shifted Gamma distribution is

$$pdf: f(t) = \begin{cases} \dfrac{\lambda^p}{\Gamma(p)} \cdot (t-a)^{p-1} \cdot e^{-\lambda \cdot (t-a)} & for \ \ t \geq a \\ 0 & else \end{cases} \tag{1}$$

with $\lambda = \dfrac{E(t-a)}{Var(t)}$ and $p = \dfrac{E^2(t-a)}{Var(t)}$ (2)

TRAVEL TIME ON A ROUTE (SECTION) CONSISTING OF SEVERAL LINKS

The travel time on a route (section) consisting of several links can be determined based on the travel times of the links (see Figure 2).

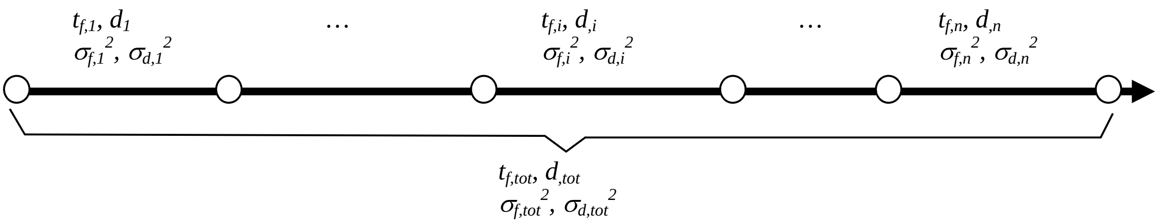

Figure 2 - Composition of the travel times and the variance over a route with several links

In general, the travel time t_T within a link can be considered as a superposition of the free flow travel time t_f and the delay d within the link. The free flow travel time t_f depends on the length L of the link and the free flow speed v_0. The delay d is a function of the flow rate q and the capacity c of the considered link. Thus, the following applies:

$$t_T = t_f + d \tag{3}$$

Here, t_T , t_f and d are regarded as random variables. For example, the travel time of a single link can be expressed by the BPR function:

$$t_T = t_f \cdot \left[1 + \alpha \cdot \left(\frac{q}{c} \right)^\beta \right] = t_f + t_f \cdot \alpha \cdot \left(\frac{q}{c} \right)^\beta \tag{4}$$

with
t_T = link travel time
t_f = free flow link travel time
q = link flow rate
c = link capacity

The coefficients α and β can be set to commonly used default values 0.15 and 4.

According to Eq. (4), the delay within the link is

$$d = t_f \cdot \alpha \cdot \left(\frac{q}{c} \right)^{\beta} \tag{5}$$

The travel time t_f in free flow conditions corresponds to the reciprocal of the free flow speed v_0. It can be considered as Gamma distributed. The delay d caused by the traffic flow q is approximately equal to the waiting time from the queuing theory. It can be described by an exponential or a Gamma distribution. The total travel time t_T as the sum of the free flow travel time t_f and the delay d can be considered as Gamma distributed as well.

For a route consisting of n links, the total travel time of the route can be calculated as (see Figure 2):

$$t_{T,tot} = \sum_{i=1}^{n} t_{T,i} = \sum_{i=1}^{n} (t_f + d_i) \tag{6}$$

with $t_{T,tot}$ = total travel time of the route
$t_{T,i}$ = travel time of the link i
$t_{f,i}$ = free flow travel time of the link i
d_i = delay of the link i

For the individual links, the free flow travel time t_f and the delay d within the links are considered to be either normal or Gamma distributed. Thus, according to the theory of statistics, the variance of the travel time $\sigma_{T,tot}^2$ over the entire route is equal to the sum of the variances of all links $\sigma_{T,i}^2$ if the individual links are considered as independent of each other. That is,

$$\sigma_{T,tot}^2 = \sum_{i=1}^{n} \sigma_{T,i}^2 = \sum_{i=1}^{n} (\sigma_{f,i}^2 + \sigma_{d,i}^2) \tag{7}$$

with $\sigma_{T,tot}^2$ = variance of the travel time for the entire route
$\sigma_{T,i}^2$ = variance of the travel time of link i
$\sigma_{f,i}^2$ = variance of the free flow travel time of link i
$\sigma_{d,i}^2$ = variance of the delay of link i

The travel time t_T and its components t_f and d from two adjacent links are not always independent of each other. In particular, the delays d of two adjacent links can be closely correlated with each other because they are usually functions of the same traffic flow rate q. In case of dependent adjacent links we have:

$$\begin{aligned} \sigma_{T,tot}^2 &= \sum_{i=1}^{n} \sigma_{T,i}^2 + 2\sum_{i=1}^{n-1} \left(k_{T,i,i+1} \cdot \sigma_{T,i} \cdot \sigma_{T,i+1} \right) \\ &= \sum_{i=1}^{n} \left(\sigma_{f,i}^2 + \sigma_{d,i}^2 \right) + 2\sum_{i=1}^{n-1} \left(k_{f,i,i+1} \cdot \sigma_{f,i} \cdot \sigma_{f,i+1} + k_{d,i,i+1} \cdot \sigma_{d,i} \cdot \sigma_{d,i+1} \right) \end{aligned} \tag{8}$$

with $k_{T,i,i+1}$ = correlation coefficient of travel time of two adjacent links
$k_{f,i,i+1}$ = correlation coefficient of free flow travel time of two adjacent links
$k_{d,i,i+1}$ = correlation coefficient of delay of two adjacent links

The values of $k_{T,i,i+1}$, $k_{f,i,i+1}$ and $k_{d,i,i+1}$ are usually very small. Normally they can be neglected ($k \approx 0$) for reasons of simplification.

The total travel time $t_{T,tot}$ of the route must also correspond to a Gamma-like distribution. For very large n ($n > 20$), it can be approximated by a normal distribution (law of large numbers). The values of the here listed times t_T, t_f, d and their variances σ_T^2, σ_f^2, σ_d^2 can be modelled theoretically for the individual links. They can also be determined directly by measurements or simulations. Here t_f is only dependent on the road type of the link and d on the road type and the flow rate. The total travel time of the route $t_{T,tot}$ and its variance $\sigma_{T,tot}^2$ can be determined by GPS measurements or license plate recognition method. By comparing the variance of the total route travel time $\sigma_{T,tot}^2$ with the variances $\sigma_{T,i}^2$ (or $\sigma_{f,i}^2$ and $\sigma_{d,i}^2$) of the individual links, the correlation coefficient of the travel time $k_{T,i,i+1}$ (or $k_{f,i,i+1}$ and $k_{d,i,i+1}$) of two adjacent links can be estimated according to Eq. (8).

The variance of the travel time σ_T^2 provides a measure of the reliability or unreliability of travel time. It is a function of the flow rate q. The relationship can also be observed in reality. If the travel times and their variances of the individual links are known, the travel time and the variance of the total route can be calculated by summation. With the calculated total travel time $t_{T,tot}$, the total variance $\sigma_{T,tot}^2$, the distribution function of the total travel time (e.g. a Gamma distribution or approximately a normal distribution), and also the required percentile of the total travel time can be determined.

In order to investigate the behavior of the total route travel time in relation to the travel times on the individual links, N links with a unit length of $L = 1$ and identical travel time t_T and variance σ_T^2 are considered. This gives now

$$t_{T,tot} = \sum_{i=1}^{N} t_{T,i} = N \cdot t_T \tag{9}$$

$$\begin{aligned} \sigma_{T,tot}{}^2 &= \sum_{i=1}^{N} \sigma_{T,i}{}^2 + 2\sum_{i=1}^{N-1}\left(k_{T,i,i+1} \cdot \sigma_{T,i}{}^2\right) \\ &= N \cdot \sigma_T{}^2 + 2(N-1) \cdot k_T \cdot \sigma_T{}^2 \end{aligned} \tag{10}$$

For a normal case is

$$\sigma_{T,tot}{}^2 \approx (1+2k_T) \cdot N \cdot \sigma_T{}^2 \quad \text{(for large number } N\text{)} \tag{11}$$

and

$$\sigma_{T,tot} \approx \sqrt{(1+2k_T) \cdot N} \cdot \sigma_T = \sqrt{1+2k_T} \cdot \sigma_T \cdot \sqrt{N} \tag{12}$$

N can be interpreted as the total length of the route or the number of links on the route under consideration. That is,

$$\begin{aligned} \sigma_{T,tot} &\approx \sqrt{1+2k_T} \cdot \sigma_T \cdot \sqrt{N} = \sqrt{1+2k_T} \cdot \sigma_T \cdot \sqrt{\frac{t_{T,tot}}{t_T}} \\ &= \sqrt{1+2k_T} \cdot \frac{\sigma_T}{\sqrt{t_T}} \cdot \sqrt{t_{T,tot}} \end{aligned} \tag{13}$$

This gives then

$$\sigma_{T,tot} = K \cdot \sqrt{t_{T,tot}} \tag{14}$$

with $K = \sqrt{1+2k_T} \cdot \frac{\sigma_T}{\sqrt{t_T}} = const.$

For the relationship between the standard deviation and the mean value of travel time, the following normalized relationship exists:

$$\frac{\sigma_{T,tot}}{\sqrt{t_{T,tot}}} = \sqrt{1+2k_T} \cdot \frac{\sigma_T}{\sqrt{t_T}} = \text{const.} \tag{15}$$

or

$$\frac{\sigma^2{}_{T,tot}}{t_{T,tot}} = (1+2k_T) \cdot \frac{\sigma^2{}_T}{t_T} = \text{const.} \tag{16}$$

Because the variation of the free flow travel time is very small, it can be neglected for simplification. Thus, using $\sigma_f = 0$, Eqs. (9) and (10) become

$$t_{T,tot} = \sum_{i=1}^{N} t_{T,i} \approx N \cdot t_T \tag{17}$$

$$\begin{aligned} \sigma_{T,tot}{}^2 &= \sum_{i=1}^{n}\left(\sigma_{d,i}{}^2\right) + 2\sum_{i=1}^{n-1}\left(k_{d,i,i+1} \cdot \sigma_{d,i} \cdot \sigma_{d,i+1}\right) \\ &\approx (1+2k_d) \cdot N \cdot \sigma_d{}^2 \end{aligned} \tag{18}$$

Thus,

$$\begin{aligned} \sigma_{T,tot} &\approx \sqrt{1+2k_d} \cdot \sigma_d \cdot \sqrt{N} = \sqrt{1+2k_d} \cdot \frac{\sigma_d \cdot}{\sqrt{d}} \sqrt{N \cdot d} \\ &= \sqrt{1+2k_d} \cdot \frac{\sigma_d}{\sqrt{d}} \cdot \sqrt{N \cdot \left(t_T - t_f\right)} = \sqrt{1+2k_d} \cdot \frac{\sigma_d}{\sqrt{d}} \cdot \sqrt{t_{T,tot} - t_{f,tot}} \\ &= K_2 \cdot \sqrt{t_{T,tot} - t_{f,tot}} \end{aligned} \tag{19}$$

with $K_2 = \sqrt{1+2k_d} \cdot \frac{\sigma_d}{\sqrt{d}}$ (20)

Eq. (19) is only defined for $t_T \geq t_f$. The corresponding probability distribution of the total travel time $t_{T,tot}$ is then a shifted Gamma distribution or a normal distribution. K_2, σ_d, and d can be measured in the field or estimated by simulation. Then, the covariance coefficient k_d can be calculated as

$$k_d = \frac{1}{2}\left[\left(\frac{K_2}{\sigma_d}\right)^2 \cdot d - 1\right] \tag{21}$$

In the literature, a linear relationship between the standard deviation $\sigma_{T,tot}$ and the mean travel time $t_{T,tot}$ is often proposed. That is,

$$\sigma_{T,tot} = a + b \cdot t_{T,tot} \tag{22}$$

with a, b = regression parameters

This linear function is unreasonable, since it contradicts the theoretical basis derived above. According to Eq. (19), the standard deviation $\sigma_{T,tot}$ is a concave function of the total travel time $t_{T,tot}$ (cf. Figure 3 for specific route with a certain free flow travel time $t_{f,tot}$, Source: Hellinga, 2011). For the special shown in Figure 3, the relationship is exactly a square function. Using the data depicted in Figure 3, Eq. (19) becomes

$$\sigma_{T,tot} = K_2 \cdot \sqrt{t_{T,tot} - t_{f,tot}} = 3.52 \cdot \sqrt{t_{T,tot} - 13.6} \qquad (23)$$

with $K_2 = 3.52$ and $t_{f,tot} = 13.6$. The corresponding coefficient of determination is R^2=0.9349. The fitting goodness is better than the linear regression with R^2=0.8908.

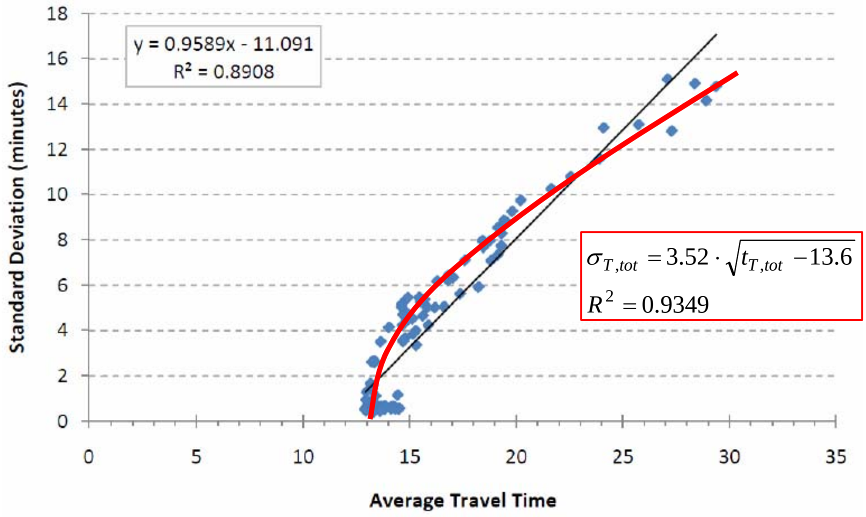

Figure 3. Dependence of the standard deviation $\sigma_{T,tot}$ from the travel time $t_{T,tot}$ (Data Highway Den Haag- Utrecht, Source: Hellinga, 2011)

That is, from Eq. (19), the total travel time of the specific route may have a shifted Gamma distribution (cf. Eq. (1)) with the parameters

$$a = 13.6,\ \lambda = \frac{E(t-a)}{Var(t)} = K_2{}^2 = 12.39 \ , \quad \text{and}$$

$$p = \frac{E^2(t-a)}{Var(t)} = K_2{}^2 \cdot (t_{T,tot} - a) = 12.39 \cdot (t_{T,tot} - 13.6)$$

Dividing both sides of Eq. (19) by the total free flow travel time $t_{f,tot}$ yields

$$\begin{aligned} SDTTI &= \frac{\sigma_{T,tot}}{t_{f,tot}} \approx \frac{K_2}{\sqrt{t_{f,tot}}} \sqrt{\frac{t_{T,tot} - t_{f,tot}}{t_{f,tot}}} \\ &= \frac{K_2}{\sqrt{t_{f,tot}}} \sqrt{TTI - 1} = K_3 \sqrt{TTI - 1} \end{aligned} \qquad (24)$$

with $SDTTI$ = standard deviation of TTI

$$TTI = \frac{\sigma_{T,tot}}{t_{f,tot}} = \text{travel time index}$$

$$K_3 = \frac{K_2}{\sqrt{t_{f,tot}}} \tag{25}$$

Again, this is a square function. This form of function using TTI as argument is also found in many other investigations (cf. Figure 4, Source: TRB SHRP2, 2013). Using the data depicted in Figure 4 (TRB SHRP2, 2013), Eq. (24) becomes

$$SDTTI = K_3\sqrt{TTI-1} = 0.6917\cdot\sqrt{TTI-1} \tag{26}$$

with K_3 =0.6717 and a corresponding coefficient of determination R^2=0.7380.

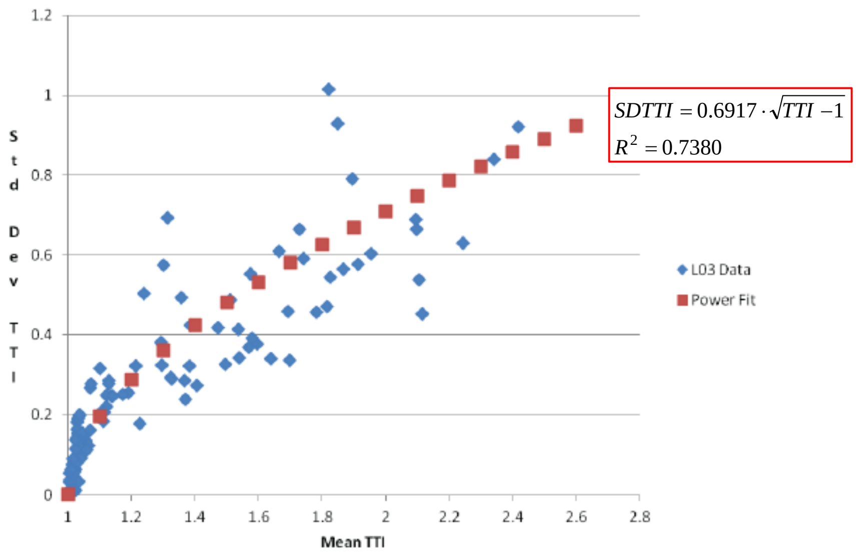

Figure 4. Dependence of the standard deviation of travel time SDTTI on the travel time index TTI (Source: TRB SHRP2, 2013; TTI = Travel Time Index)

Then the expression of the total standard deviation of the route is

$$\begin{aligned}\sigma_{T,tot} &= SDTTI\cdot t_{f,tot} = K_3\cdot t_{f,tot}\cdot\sqrt{TTI-1}\\ &= K_3\cdot\sqrt{t_{f,tot}}\cdot\sqrt{TTI\cdot t_{f,tot}-t_{f,tot}} = K_3\cdot\sqrt{t_{f,tot}}\cdot\sqrt{t_{,T,tot}-t_{f,tot}}\\ &= 0.6917\cdot\sqrt{t_{f,tot}}\cdot\sqrt{t_{,T,tot}-t_{f,tot}}\end{aligned} \tag{27}$$

Again, the parameters of the shifted Gamma distributed (cf. Eq. (1)) total travel time of the route can be obtained from Eq. (27):

$$\lambda = \frac{E(t-a)}{Var(t)} = K_3^{\ 2}\cdot t_{f,tot} = 0.4784\cdot t_{f,tot} \quad \text{and}$$

$$p = \frac{E^2(t-a)}{Var(t)} = K_3^{\ 2}\cdot t_{f,tot}\cdot(t_{T,tot}-t_{f,tot}) = 0.4784\cdot t_{f,tot}\cdot(t_{T,tot}-t_{f,tot})$$

Both parameters λ und p are increasing functions of the total free flow travel time $t_{f,tot}$ and thus of total distance travelled. Using Eq. (27), the standard deviation $\sigma_{T,tot}$ for total travel time a route with arbitrary length $t_{f,tot}$ of and the parameters λ and p of the corresponding Gamma distribution of the total travel time can be calculated directly.

For the specific route depicted in Figure 3, the formula for standard deviation of TTI is

$$SDTTI = K_3 \sqrt{TTI - 1} = \frac{K_2}{\sqrt{t_{f,tot}}} \sqrt{TTI - 1}$$
$$= \frac{3.52}{\sqrt{13.6}} \cdot \sqrt{TTI - 1} = 0.9544 \cdot \sqrt{TTI - 1} \quad (28)$$

with K_3 =0.9544. The value of K_3 here is different to the value in Eq. (26). In general, the value K_3 is road-type specific. It should be calibrated with field data.

CONCLUSIONS

For a whole route consisting of several individual links, the variance of the total route travel time can be calculated as the superposition of the variances of the individual links. Using the variance or standard deviation of travel time as an indicator of reliability, the reliability of a route or a network can be assembled from the reliability of the individual links. If the reliability (here represented by the variance or standard deviation of the travel time) for each type of road – empirically and theoretically – can be determined, the reliability of a route or a network can be easily estimated according to the proposed model. It can be found that the total standard deviation is a concave function of the total travel time. For the special case of a route with several unique links, the relationship is a square function. A model (Eq. (27)) for estimating the standard deviation of travel time within a route and the parameters of the corresponding travel time distribution is given.

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