MODELING TRAVEL TIME FOR RELIABILITY ANALYSIS IN A FREEWAY NETWORK

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ABSTRACT

Travel time reliability is a new way of looking at congestion and unpredictable variation of travel time. The variance or standard deviation of travel time can be used as an indicator for investigating the reliability of a road network. In this paper, a mathematical model dealing with the standard deviation of the total travel time within a freeway network is presented. It is shown that under some suitable assumptions, the variance of the total route travel time can be calculated as a superposition of the variances of travel time in single links or bottlenecks if the variances of total travel time and correlation coefficients between two consecutive links or bottlenecks are known. The variances and correlation coefficients can be calibrated by measurements or simulation. Using a shifted Gamma distribution to describe the total travel time, percentiles and thus the reliability of the total travel time can be quantitatively investigated. The model is calibrated with field data from US and European freeways. As a result, a procedure for estimating travel time reliability in a freeway network is recommended.

INTRODUCTION

Reliability measures are increasingly applied for assessing congestion and unpredictable variations of travel time. Reliability is one of the key indicators for the performance and service quality of transport systems, particularly for commercial vehicle traffic. Many researchers have been working on this subject (e.g. *1-7*). First of all, travel time reliability is a perception of travelers. Travel time reliability significantly influences the choice of routes, departure times, and trip link chains (*8-11*). Bates et al. (*12*) showed that one minute reduction in the standard deviation of travel time and two minutes reduction in the actual travel time can be considered equivalent. With increasing attention on travel time reliability, different definitions and measures of travel time reliability were proposed (*3, 13*). These measures relate to properties of the day-to-day, within-day, or vehicle-to-vehicle travel time distributions, particularly to their shape. There are many candidate measures having very little correlation among themselves (*3*). Bogers (*14*) concludes that the most suitable measure for travel time reliability depends on what kind of effects of reliability is evaluated. This inconsistency leads to different assessment criteria used by policy evaluations and causes ambiguous evaluations.

In the remainder of the paper, the variance or standard deviation is used for defining travel time reliability. The variance or standard deviation describes unambiguously the day-to-day variation of travel time. The day-to-day or within-day variation of travel time can be caused by unexpected weather conditions, work zones, incidents as well as the vehicle-to-vehicle variation of travel time due to the stochastic nature of traffic flow and the variation of capacity (*15-18*). These random events and influences can lead to traffic congestion and increase the standard deviation of travel time and thus the unreliability of the road network.

For a single freeway link with homogeneous characteristics, the free-flow travel time and its standard deviation can be easily obtained either by measurements or by existing models. Congestion within a freeway link can be considered as a result of a bottleneck within the link. It is also possible to estimate the distribution of delays occurring at such bottlenecks. In a network consisting of several consecutive components such as freeway links and bottlenecks, the total travel time can be considered as a superposition of free-flow travel times of the links and delays of the bottlenecks. Thus, the total travel time of a route is the sum of all free-flow travel times of the links and all delays at the bottlenecks. However, the standard deviation of the total travel time of a route is not equal to the sum of the standard deviations of travel times or delays within the single links. Given the distribution of the total travel time, the percentiles and thus the reliability of the total travel time can be quantitatively investigated.

Many studies related to fitting the travel time distribution from observed travel time data were published (e.g. *19-25*). Wardrop (*19*), for example, suggested that travel times follow a skewed distribution. Herman and Lam (*20*) proposed either the Gamma or lognormal distribution to represent the travel time distribution. Richardson and Taylor (*21*) found that the observed travel time might be fitted by a lognormal distribution. Polus (*22*) concluded that the Gamma distribution was better than the normal or lognormal distribution, and Al-Deek and Emam (*23*) proposed the Weibull distribution to fit observed travel times. Van Lint et al. (*3*) depicted travel time distributions with four different shapes based on traffic conditions (free-flow, congestion onset, congestion, and congestion dissolve). Pu (*24*) concluded that these four shapes of travel time distributions are similar to those of the lognormal distribution. Susilawati et al. (*25*) proposed the Burr Type XII distribution for travel time variability on urban roads. Based on the distributions of travel times, a large number of travel time reliability measures was proposed by previous researchers (e.g. *3, 12, 26*-*30*). If the distribution of travel time is known, the measures of reliability can be defined correspondingly. Other researchers studied the relation between the mean travel time per unit of distance and the standard deviation of travel time (e.g. *30*-*33*) as well as the optimal path finding in stochastic networks (e.g. *34*-*35*). Kim and Mahmassani (*36*) proposed a compound Gamma representation for modeling travel time variability in a road network. They found the compound Gamma distribution is best representing different variability dimensions in connection with vehicle-to-vehicle and day-to-day variability.

Wu and Geistefeldt (*37*) presented a mathematical model dealing with the standard deviation of the total travel time within a freeway route. In general, the distribution of the free-flow travel time of links and the distribution of delays at bottlenecks can be described either by an exponential, a normal, an Erlang, or a Gamma distribution (among other possible distributions). The parameters of these distributions can be calibrated by measurements or simulation. The variance of the total travel time of a route can be calculated as the sum of the variances of the single links in case that the travel times and the delays are statistically independent. In reality, the independency between the consecutive links may not exist. In this case, the variance of the total travel time of a route can also be estimated if the correlation coefficient between two consecutive links is known. Again, this correlation coefficient can be calibrated by measurements or simulation. Once the variance of the travel time is known, the standard deviation is also known.

Wu and Geistefeldt (*37*) suggested the shifted Erlang or Gamma distribution for describing the travel time. The Erlang distribution is a special case of Gamma distribution. In the remainder, the Gamma distribution is applied as a generalized solution. Because of the special property of the Gamma distribution, the variance of the total route is equal to the sum of the variances of the single links. The travel time of the total route is considered as Gamma distributed again. In order to account for the lower limit of travel time, a shifted Gamma distribution is used.

In the following, a normalized shifted Gamma distribution is introduced to describe the travel time, its percentiles and thus the travel time reliability. The model is used to estimate the variance or standard deviation of the travel time on a route consisting of several links. The empirical calibration of the model for estimating the variance or standard deviation of travel time is demonstrated for some examples. For applications, a procedure to estimate the reliability of a freeway route or network is given.

NORMALIZED SHIFTED GAMMA TRAVEL TIME DISTRIBUTION

The average travel time includes expected and unexpected delays. Unexpected delays lead to a variation in travel time. This variability can be quantified by various characteristics of the distribution of travel time over a certain time period, such as the standard deviation of the travel time distribution. The travel time distribution has a common shape with positive skewness. This shape can be represented by a shifted Gamma distribution. Using different combinations of mean value and standard deviation, different shapes of the travel time distribution for different traffic conditions (free-flow, congestion onset, congestion, and congestion dissolve, see (*3*)) can be reproduced.

The probability density function of the shifted Gamma distribution is:

$$f_\Gamma(t) = \begin{cases} \dfrac{\lambda^p}{\Gamma(p)} \cdot (t-a)^{p-1} \cdot e^{-\lambda\cdot(t-a)} & \text{for } t \geq a \\ 0 & \text{else} \end{cases} \tag{1}$$

$$\text{with } \lambda = \frac{E(t-a)}{Var(t-a)} = \frac{E(t-a)}{Var(t)} \text{ and } p = \frac{E^2(t-a)}{Var(t-a)} = \frac{E^2(t-a)}{Var(t)} \tag{2}$$

Figure 1 shows an example of the shifted Gamma distribution. When describing the travel time distribution, the parameter a represents the free-flow travel time t_f and E(t–a) the mean delay d of the link or route under consideration. The shifted Gamma distribution has a positive skewness, which is also an important property of the travel time distribution.

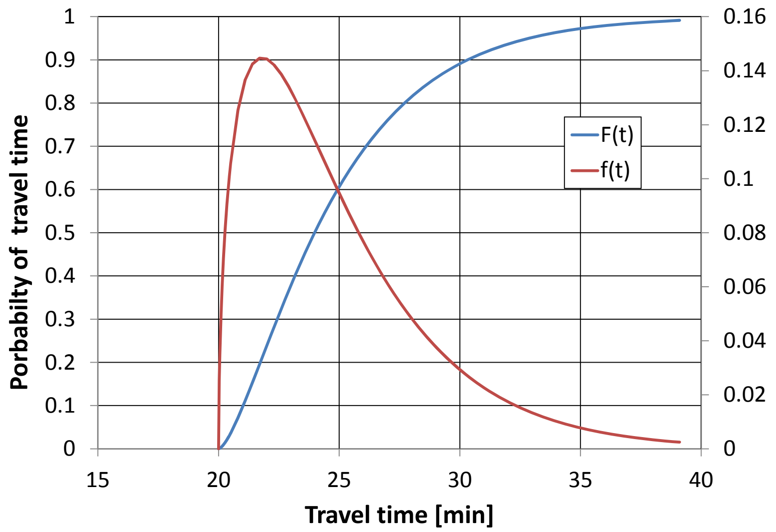

FIGURE 1 Shifted Gamma distribution with a = t_f = 20 min, E(t-a) = E(d) = 5 min, and VAR(t) = 16 min^2.

Because the parameter $a = t_f$ is a constant. Eq. (1) can be simplified for the delay d by substituting $t - a = d$. That is, the delay obeys a Gamma distribution of the form

$$f_\Gamma(d) = \begin{cases} \dfrac{\lambda^p}{\Gamma(p)} \cdot d^{p-1} \cdot e^{-\lambda\cdot d} & \text{for } d \geq 0 \\ 0 & \text{else} \end{cases} \tag{3}$$

$$\text{with } \lambda = \frac{E(d)}{Var(d)} \text{ and } p = \frac{E^2(d)}{Var(d)} = \lambda \cdot E(d) \tag{4}$$

The coefficient of variation of this Gamma distribution for delay d is

$$CV_d = \frac{SD(d)}{E(d)} = \frac{\sqrt{Var(d)}}{E(d)} = \frac{1}{\sqrt{p}} \tag{5}$$

Thus, the parameters p and λ can be expressed as

$$p = \frac{1}{CV_d^{\ 2}} \quad \text{and} \quad \lambda = \frac{p}{E(d)} = \frac{1}{CV_d^{\ 2} \cdot E(d)} \tag{6}$$

Eq. (3) can be normalized for $d^* = d / E(d)$. Thus,

$$f_\Gamma^{\ *}(d^*) = \begin{cases} \dfrac{p^p}{\Gamma(p)} \left(d^*\right)^{p-1} \cdot e^{-p \cdot d^*} & \text{for } d^* \geq 0 \\ 0 & \text{else} \end{cases} \tag{7}$$

Eq. (7) is a function of only one parameter $p = 1/CV_d^{\ 2}$. It is independent of the mean delay E(d). The corresponding probability distribution function of Eq. (7) is then

$$F_\Gamma^{\ *}(d^*) = \int_0^{d^*} f_\Gamma^{\ *}(x)dx = \int_0^{d^*} \frac{p^p}{\Gamma(p)} x^{p-1} \cdot e^{-p \cdot x} dx \tag{8}$$

This function cannot be calculated easily. However, spreadsheet software (i.e. EXCEL) can be used in order to establish a graph (Figure 2). This graph has only one parameter, the coefficient of variation of the delay CV_d.

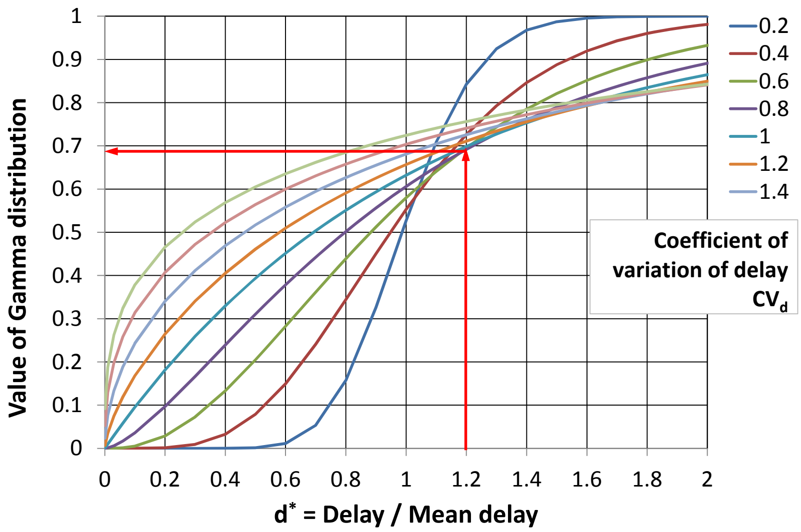

FIGURE 2 Graph for the normalized Probability Distribution Function (PDF) of the shifted Gamma distribution. Example: d^* = 1.2, CV_d = 0.8 → $F(d^*)$ = 0.69.

For the special case that t_f is considered as deterministic, the following relationships are always true:

$$E(t_T) = t_f + E(d) \quad (9)$$

$$t_{T,median} = t_f + d_{median} \quad (10)$$

$$t_{T,percentile} = t_f + d_{percentile} \quad (11)$$

To reduce the risk of being late at the destination, the driver needs to allow rather more time than the mean travel time. The travel time unreliability increases with the widening (the longer the tail) of the travel time distribution. Hence, indicators representing travel time reliability are usually defined based on parameters of the travel time distribution.

Using Figure 2, the probability of trips on time can be obtained given the value of CV_d and a permitted delay $d_{perm} = d^* \cdot E(d)$. For the example depicted in Figure 1 we have $t_f = 20$ min, $E(t - a) = E(d) = 5$ min, and $Var(d) = 16$ min^2. The probability of trips on time for permitted delay $d_{perm} = 6$ min is 0.69 ($d^* = d^* \cdot E(d) = 6/5 = 1.2$, $CV_d = SD(d)/E(d) = 16^{0.5}/5 = 0.8$). Thus, the probability of trips on time for a travel time $t_T = t_f + d_{perm} = 20 + 6 = 26$ min is 0.69.

The reverse function of Eq. (8) yields the permitted delay given a value of CV_d and a probability of trips on time. A graph can also be constructed for this function (Figure 3).The curve for a probability of 0.5 for trips being on time indicates the median value of the delay, d_{median}.

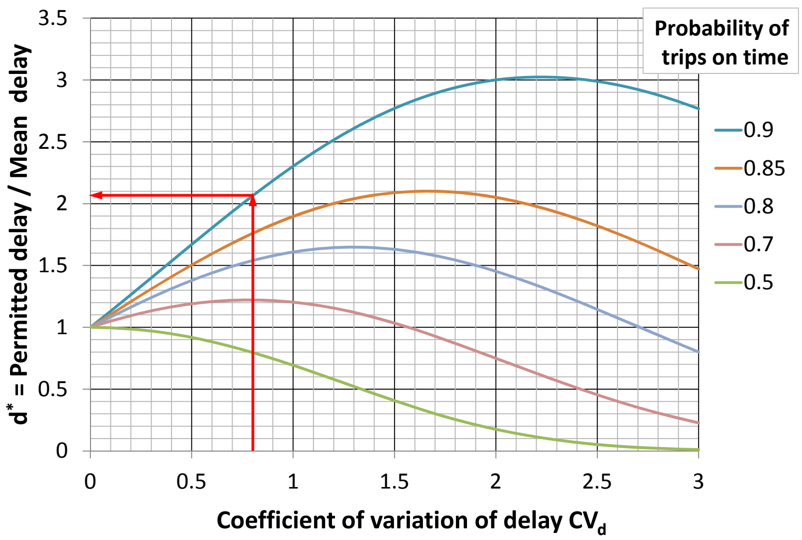

FIGURE 3 Graph for permitted delay. Example: CV_d = 0.8, $F(d^*)$ = 0.9 $\rightarrow$ d^*=2.06.

For the same example above, if a probability of 0.9 for trips being on time is required, a delay $d_{perm} = d^* = E(d) = 2.06 \cdot 5 = 10.3$ min or a travel time $t_{T,perm} = t_f + d_{perm} = 30 + 10.3 = 30.3$ min must be permitted. The median travel time is $t_{T,median} = t_f + d_{median} = 20 + 0.8 \cdot 5 = 24$ min. It is less than the mean travel time ($t_f + E(d) = 20 + 5 = 25$ min).

For calculating the travel time, a suitable model should be used. This can be obtained from the literature. For example, a well-known function for calculating the travel time is provided by the Bureau of Public Roads (*38*). The BPR function is

$$t_T = t_f \cdot \left[1 + \alpha \cdot x^\beta\right] = t_f + t_f \cdot \alpha \cdot x^\beta \tag{12}$$

with t_T = link travel time
t_f = free-flow link travel time
x = degree of saturation
= q / c
q = link flow rate
c = link capacity

The coefficients α and β can be set to commonly used default values 0.15 and 4. They can also be calibrated against field data. According to Eq. (12), the delay within a link is

$$d = t_f \cdot \alpha \cdot x^\beta \tag{13}$$

The standard deviation within a route is normally unknown. In the following section, a model for estimating the standard deviation of route travel time is introduced.

STANDARD DEVIATION OF ROUTE TRAVEL TIME

Using the normalized shifted Gamma distribution presented in the previous section, the travel time, its percentiles and thus the reliability can be easily expressed as a simple function of the coefficient of variation of the travel time. Here, we concentrate on how the coefficient of variation of the travel time can be estimated. For doing this, a model for estimating the standard deviation of the total travel time within a freeway route (*37*) is reintroduced in the following.

The travel time on a route consisting of several links can be determined based on the travel times of the links (see Figure 4).

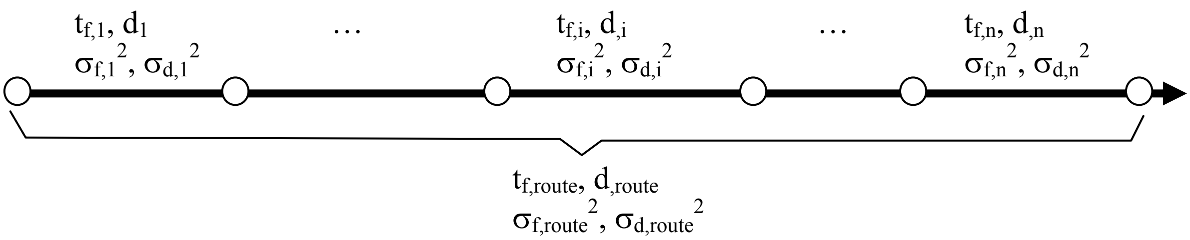

FIGURE 4 Composition of the travel times and their variances over a route with several links.

In general, the travel time t_T within a link can be considered as a superposition of the free-flow travel time t_f and the delay d within the link. The free-flow travel time t_f depends on the length L of the link and the free-flow speed v_f. The delay d is a function of the demand flow q and the capacity c of the considered link. Thus, the following applies.

$$t_T = t_f + d \tag{14}$$

Here, t_T, t_f, and d are defined as the means of the regarded random variables.

The travel time t_f in free-flow conditions corresponds to the reciprocal of the free-flow speed v_f and can be considered as Gamma distributed. The delay d caused by the demand flow q is approximately equal to the waiting time from queuing theory. It can be described by an exponential or a Gamma distribution. The total travel time t_T as the sum of the free-flow travel time t_f and the delay d can be considered as Gamma distributed as well.

For a route consisting of n links, the total travel time of the route can be calculated as (see Figure 4):

$$t_{T,route} = \sum_{i=1}^{n} t_{T,i} = \sum_{i=1}^{n} (t_f + d_i) \tag{15}$$

with $t_{T,route}$ = total travel time of the route
$t_{T,i}$ = total travel time of the link i
$t_{f,i}$ = free-flow travel time of the link i
d_i = delay of the link i

For the individual links, the free-flow travel time t_f and the delay d within the links are considered to be either exponential or Gamma distributed. Thus, according to the theory of statistics, the variance of the travel time $\sigma_{T,route}^2$ over the entire route is equal to the sum of the variances of all links $\sigma_{T,i}^2$ in case the individual links are independent of each other. That is,

$$\sigma_{T,route}^2 = \sum_{i=1}^{n} \sigma_{T,i}^2 = \sum_{i=1}^{n} (\sigma_{f,i}^2 + \sigma_{d,i}^2) \tag{16}$$

with $\sigma_{T,route}^2$ = variance of the total travel time for the entire route
$\sigma_{T,i}^2$ = variance of the total travel time of link i
$\sigma_{f,i}^2$ = variance of the free-flow travel time of link i
$\sigma_{d,i}^2$ = variance of the delay of link i

The travel time t_T and its components t_f and d from two adjacent links are not always independent of each other. In particular, the delays d of two adjacent links can be closely correlated with each other because they are usually functions of the same traffic demand q. In case of dependent adjacent links we have:

$$\begin{aligned}\sigma_{T,route}^2 &= \sum_{i=1}^{n} \left(\sigma_{T,i}^2 + 2k_{T,i,i+1} \cdot \sigma_{T,i} \cdot \sigma_{T,i+1}\right) \\ &= \sum_{i=1}^{n} \left[\sigma_{f,i}^2 + \sigma_{d,i}^2 + 2\left(k_{f,i,i+1} \cdot \sigma_{f,i} \cdot \sigma_{f,i+1} + k_{d,i,i+1} \cdot \sigma_{d,i} \cdot \sigma_{d,i+1} + k_{f,d,i}\sigma_{f,i} \cdot \sigma_{d,i}\right)\right]\end{aligned} \tag{17}$$

with $k_{T,i,i+1}$ = correlation coefficient of the total travel time of two adjacent links
$k_{f,i,i+1}$ = correlation coefficient of the free-flow travel time of two adjacent links
$k_{d,i,i+1}$ = correlation coefficient of the delay of two adjacent links
$k_{f,d,i}$ = correlation coefficient of free-flow travel time and the delay within a link

The value of $k_{f,d,i}$ is normally very small. The values of $k_{T,i,i+1}$, $k_{f,i,i+1}$, and $k_{d,i,i+1}$ are usually also very small if only links of sufficient lengths are considered. Normally they can be neglected ($k \approx 0$) for simplification.

The total travel time $t_{T,route}$ of the route must also correspond to a Gamma-like distribution. Moschopoulus (*39*) gives an exact expression of this distribution. However, this expression is very complex and an explicit solution is not available for practical uses. Thus, a shifted Gamma distribution is utilized also for the total travel time $t_{T,route}$ as a simplification. The values of the here listed times t_T, t_f, d and their variances σ_T^2, σ_f^2, σ_d^2 can be modelled theoretically for the individual links. They can also be determined directly by measurement or simulation. Here t_f is only dependent on the road type of the link and d on the road type and the demand flow. The total travel time of the route $t_{T,route}$ and its variance $\sigma_{T,route}^2$ can be determined e.g. by GPS measurements or license plate recognition. By comparing the variance of the total route travel time $\sigma_{T,route}^2$ with the variances $\sigma_{T,i}^2$ (or $\sigma_{f,i}^2$ and $\sigma_{d,i}^2$) of the individual links, the correlation coefficients of the travel time within a link or between two adjacent links can be estimated according to Eq. (17).

The variance of the travel time σ_T^2 provides a measure of the reliability or unreliability of travel time. It is a function of the demand flow q. The relationship can also be observed in reality. If the travel times and their variances of the individual links are known, the travel time and the variance of the total route can be calculated by summation. With the calculated total travel time $t_{T,route}$, the total variance $\sigma_{T,route}^2$, the distribution function of the total travel time (e.g. a Gamma distribution or approximately a normal distribution) and also the required percentile of the total travel time can be determined.

In order to investigate the behavior of a link travel time in relation to the length of the link, N sub-segments s with a unit length of L = 1 and identical travel time $t_{T,s}$ and variance $\sigma_{T,s}^2$ are considered within the links. This gives now

$$t_T = \sum_{j=1}^{N} \left(t_{T,s}\right)_j = N \cdot t_{T,s} \tag{18}$$

$$\begin{aligned} \sigma_T^2 &= \sum_{j=1}^{N} \left[\left(\sigma_{T,s}^2\right)_j + 2\left(k_{T,s}\right)_{j,j+1} \cdot \left(\sigma_{T,s}^2\right)\right] \\ &= N \cdot \left(\sigma_{T,s}^2 + 2k_{T,s} \cdot \sigma_{T,s}^2\right) \\ &= (1 + 2k_{T,s}) \cdot N \cdot \sigma_{T,s}^2 \end{aligned} \tag{19}$$

with $k_{T,s} = k_{f,s} + k_{d,s} + k_{f,d,s}$ (cf. Eq. (17)). Thus,

$$\sigma_T = \sqrt{(1 + 2k_{T,s}) \cdot N} \cdot \sigma_{T,s} = \sqrt{1 + 2k_{T,s}} \cdot \sigma_{T,s} \cdot \sqrt{N} \tag{20}$$

N can be interpreted as the total length of the link or the number of sub-segments in the link under consideration. The term $(k_{T,s})_{N,N+1}$ is than the correlation coefficient of the travel time to the following link. That is,

$$\begin{aligned}\sigma_T &= \sqrt{1+2k_{T,s}} \cdot \sigma_{T,s} \cdot \sqrt{N} = \sqrt{1+2k_{T,s}} \cdot \sigma_{T,s} \cdot \sqrt{\frac{t_T}{t_{T,s}}} \\ &= \sqrt{1+2k_{T,s}} \cdot \frac{\sigma_{T,s}}{\sqrt{t_{T,s}}} \cdot \sqrt{t_T}\end{aligned} \tag{21}$$

This gives then

$$\sigma_T = K \cdot \sqrt{t_T} \tag{22}$$

with $K = \sqrt{1+2k_{T,s}} \cdot \frac{\sigma_{T,s}}{\sqrt{t_{T,s}}} = \text{const.}$

Compared to the delay caused by a bottleneck, the variation of the free-flow travel time is very small. It can be neglected for simplification. Thus,

$$\sigma_T \approx \sigma_d = K_2 \cdot \sqrt{t_T - t_f} = K_2 \cdot \sqrt{d} \tag{23}$$

with $K_2 = \sqrt{1+2k_{d,s}} \cdot \frac{\sigma_{d,s}}{\sqrt{d_s}}$ (24)

Eq. (23) can also be expressed as

$$\begin{aligned}\sigma_T &= \sigma_d \\ &= K_2 \cdot \sqrt{t_T - t_f} = K_2 \cdot \sqrt{t_f} \cdot \sqrt{\frac{t_T}{t_f} - 1} \\ &= K_2 \cdot \sqrt{t_f} \cdot \sqrt{TTI - 1}\end{aligned} \tag{25}$$

with $TTI = \frac{t_T}{t_f}$ = travel time index. Thus,

$$\begin{aligned}SD_{TTI} &= \frac{\sigma_T}{t_f} = \frac{\sigma_d}{t_f} \\ &= \frac{K_2}{\sqrt{t_f}} \cdot \sqrt{\frac{t_T}{t_f} - 1} \\ &= \frac{K_2}{\sqrt{t_f}} \cdot \sqrt{TTI - 1} = K_3 \cdot \sqrt{TTI - 1}\end{aligned} \tag{26}$$

with SD_{TTI} = standard deviation of travel time index

$TTI = \frac{t_T}{t_f}$ = travel time index (27)

$$K_3 \quad = \frac{K_2}{\sqrt{t_f}}$$

Eqs. (23) and (25) are only defined for $t_T \geq t_f$ ($TTI \geq 1$). The corresponding link travel time t_T is considered as a shifted Gamma distributed. K_2, $\sigma_{d,s}$, and d_s can be measured in the field or estimated by simulation. Then, the covariance coefficient $k_{d,s}$ can be calculated as

$$k_{d,s} = \frac{1}{2}\left[\left(\frac{K_2}{\sigma_{d,s}}\right)^2 \cdot d_s - 1\right] \tag{28}$$

Accordingly, the coefficient of variation of the link travel time is

$$CV_d = \frac{\sigma_d}{d} = \frac{K_2 \cdot \sqrt{d}}{d} = \frac{K_2}{\sqrt{d}} = \frac{K_2}{\sqrt{t_f} \cdot \sqrt{TTI - 1}} = \frac{K_3}{\sqrt{TTI - 1}} \tag{29}$$

Once the parameter K_2 (or K_3) is calibrated with field data, the covariance coefficient k and thus the dependence between the travel times in adjacent sub-segments is incorporated into K_2. Thus, for $\sigma_f = 0$, Eq. (17) for calculating route variance of travel time can be formulated as:

$$\sigma_{T,tot}{}^2 = \sigma_{d,tot}{}^2 = \sum_{i=1}^{n}\left(K_{2,i} \cdot \sqrt{d_i}\right)^2 = \sum_{i=1}^{n}\left(K_{2,i}{}^2 \cdot d_i\right) \tag{30}$$

CALIBRATION OF THE STANDARD DEVIATION FROM FIELD DATA

According to Eq. (23), the standard deviation σ_T of a link is a concave function of the link travel time t_T. For the example shown in Figure 5 (*40*), the relationship is exactly a square function. Using the data depicted in Figure 5, Eq. (23) becomes

$$\sigma_T = \sigma_d = K_2 \cdot \sqrt{t_T - t_f} = 3.52 \cdot \sqrt{t_T - 13.6} \tag{31}$$

with $K_2 = 3.52$ and t_f, = 13.6 min. The corresponding coefficient of determination is $R^2 = 0.9349$. The goodness of fit is much better than the linear regression with R^2=0.8908. This linear function is unreasonable, since it contradicts the theoretical basis derived above.

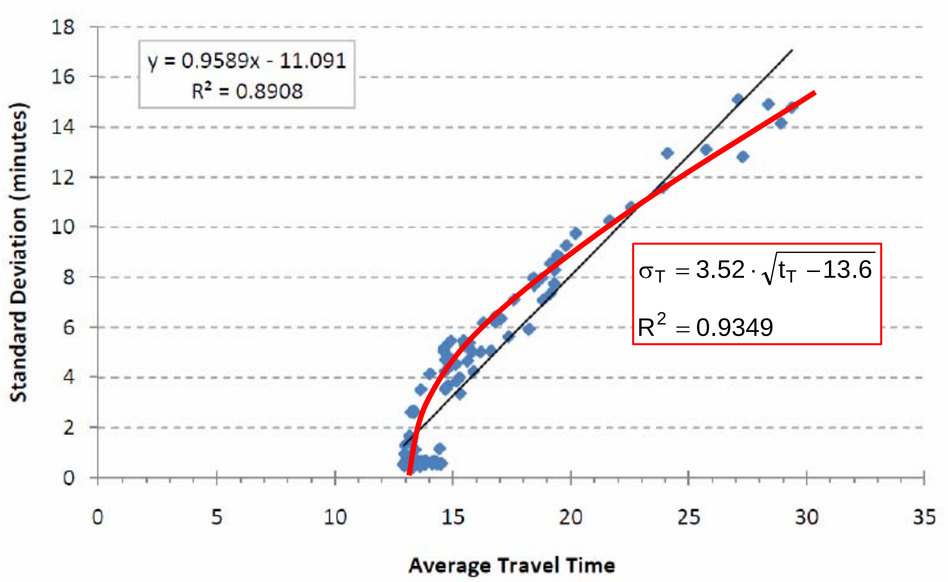

FIGURE 5 Dependence of the standard deviation σ_T from the travel time t_T (Data: Freeway Den Haag – Utrecht, Netherlands, Source: (*40*)).

Eq. (31) yields

$$SD_{TTI} = \frac{\sigma_T}{t_f} = \frac{\sigma_d}{t_f} = \frac{K_2 \cdot \sqrt{t_T - t_f}}{\sqrt{t_f}\sqrt{t_f}} = \frac{K_2}{\sqrt{t_f}} \cdot \sqrt{TTI - 1} = \frac{3.52}{\sqrt{13.6}} \cdot \sqrt{TTI - 1} = 0.95 \cdot \sqrt{TTI - 1} \tag{32}$$

That is, from Eq. (23), the total travel time of the link may have a shifted Gamma distribution (cf. Eq. (1)) with the parameters

$$a = t_f = 13.6, \; \lambda = \frac{E(t-a)}{Var(t-a)} = \frac{1}{K_2{}^2} = 0.081 \text{ and}$$

$$p = \frac{E^2(t-a)}{Var(t-a)} = \frac{(t_T - a)}{K_2{}^2} = 0.081 \cdot (t_T - 13.6) = 0.081 \cdot d$$

For using the graphs (Figure 2 and Figure 3), only the parameter CV_d is required:

$$CV_d = \frac{K_2}{\sqrt{d}} = \frac{3.52}{\sqrt{d}}$$

In Figure 6, other data examples from three bottleneck sections on a German freeway are depicted. In these data, the delays are directly measured. Thus, the relationship of standard deviation to delay can be expressed as

$$\sigma_T \approx \sigma_d \approx K_2 \cdot \sqrt{d} = K_3 \cdot \sqrt{t_f} \cdot \sqrt{d} \tag{33}$$

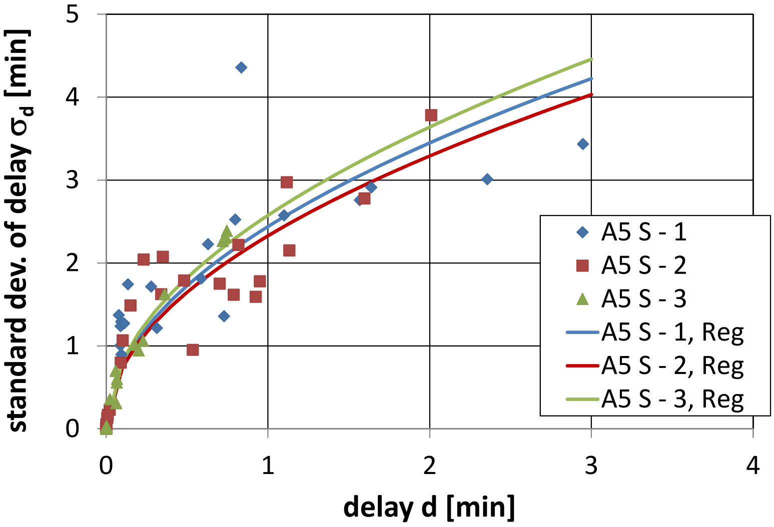

FIGURE 6 Dependence of the standard deviation $\sigma_T = \sigma_d$ from the total delay d (Data of freeway A 5, Germany, Source: (*41*))

The values of K_2 and K_3 for the examples in Germany are given in Table 1. The values of K_3 are different but comparable (cf. also the value of K_3 in Eq. (32)). That means the freeway links have different but comparable characteristics.

TABLE 1 Parameters of the example bottleneck sections in Germany.

Name	Length	Free-flow speed	Free-flow travel time	K_2	K_3	R^2
	L [km]	v_f [km/h]	t_f [min]			
A5 S - 1	9	120	4.5	2.44	1.15	0.91
A5 S - 2	14.8	120	7.4	2.33	0.86	0.92
A5 S - 3	8.8	120	4.4	2.57	1.23	0.92

Using the data depicted in Figure 7 (*42*), Eq. (26) becomes

$$SD_{TTI} = K_3 \sqrt{TTI-1} = 0.6917 \cdot \sqrt{TTI-1} \quad (34)$$

with K_3 =0.6717 and a corresponding coefficient of determination R^2 = 0.7380.

In general, the value K_3 is site-specific and depends on the prevailing traffic and control conditions as well as geometric factors such as number of lanes, distance of access points, grade, lane widths, shoulder widths, and sharpness of horizontal and vertical curves. A large K_3-value reflects the fact that the distances between the bottlenecks (i.e. distances between access points) are smaller and thus the delay per kilometer is larger.

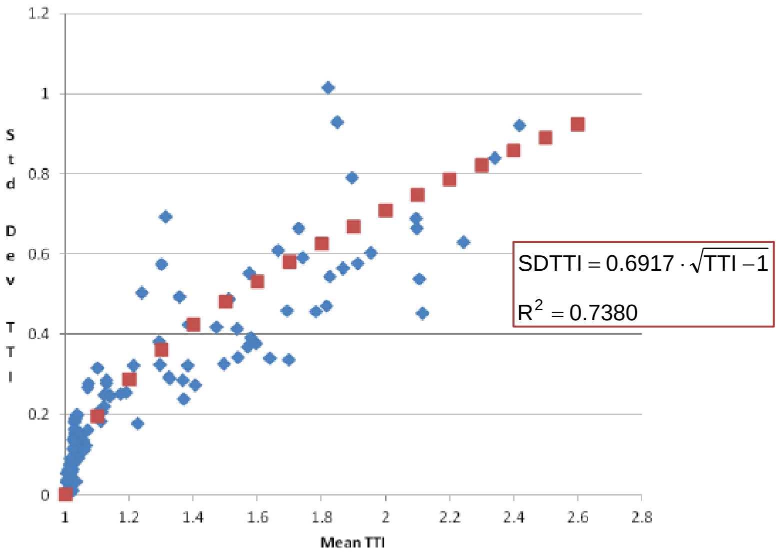

FIGURE 7 Dependence of the standard deviation of travel time SD_{TTI} on the travel time index TTI for a US freeway (Source: (*42*), TTI = Travel Time Index)

PROCEDURE FOR ESTIMATING RELIABILITY OF A ROUTE

For practical application, the following procedure can be recommended for determining the reliability in a route consisting of several homogenous links:

1. Estimation of input parameters for the single links
    a. Length L in (km)
    b. Free-flow speed v_f in (km/h)
    c. Free-flow travel time t_f in (min) ($t_f = L / v_f \cdot 60$)
    d. Parameter K_3 or parameter K_2 ($K_2 = K_3 \cdot \sqrt{t_f}$)
    e. Demand q in (veh/h)
    f. Capacity c in (veh/h)
2. Estimation of output parameters for the links
    a. Calculation of the average delay d caused by demand using a suitable model (i.e. using the BPR function, Eq. (13))
    b. Calculation of the mean travel time t_T ($t_T = t_f + d$)
    c. Calculation of the standard deviation of delay σ_d (Eq. (23) or (25))
    d. Calculation of the coefficient of variation of delay CV_d (Eq. (29))
    e. Determination of the median (d_{median}) or percentiles (d_{80} or d_{90}) of the delay using Figure 3 (or using a spreadsheet)
    f. Determination of the median ($t_{T,median}$) or percentiles ($t_{T,80}$ or $t_{T,90}$) of the travel time (Eqs. (10) and (11))
3. Estimation of output parameters for the route consisting of the links
    a. Calculation of the total average delay

$$d_{route} = \sum d$$

b. Calculation of the mean travel time

$$t_{T,route} = \sum t_T$$

c. Calculation of the standard deviation of delay

$$\sigma_{d,route} = \sqrt{\sum \sigma_d}$$

d. Calculation of the coefficient of variation of delay $CV_{d,route}$

$$CV_{d,route} = \sigma_{d,route} / \sigma_{d,route}$$

e. Determination of the median ($d_{median,route}$) or percentiles (e.g. $d_{80,route}$ or $d_{90,route}$) of the delay using Figure 3 (or using a spreadsheet)

f. Determination of the median ($t_{T,median,route}$) or percentiles (e.g. $t_{T,80,route}$ or $t_{T,90,route}$) of the travel time (Eqs. (10) and (11))

For an example calculation, we consider a route of freeway A 5 in Germany, consisting of 3 subsequent bottleneck sections. The BPR function is applied for calculating the travel time. The input parameters and the results of the example are given in Table 2.

TABLE 2 Parameters and calculation results for the example links.

	Link *i*	A5 N - 2	A5 N - 3	A5 N - 4	total (route)
parameters	Length L [km]	5.5	14.8	12.1	32.4
	FFS v_f [km/h]	120	120	120	-
	Free-flow travel time t_f [min]	2.75	7.40	6.05	16.2
	Parameter K_3	0.97	1.10	0.54	-
	Parameter K_2	1.62	3.01	1.32	-
	Demand q [veh/h]	4800	5500	5400	-
	Capacity c [veh/h]	5400	5600	5400	-
results	Delay by demand [min]	0.26	1.03	0.91	2.20
	Total mean delay d [min]	0.26	1.03	0.91	2.20
	Mean travel time t_T [min]	3.01	8.43	6.96	**18.40**
	SD of delay σ_d [min]	0.82	3.05	1.26	3.40
	CV_d [-]	3.19	2.96	1.39	1.55
	d_{median} [min]	0.00	0.01	0.43	0.83
	d_{80} [min]	0.17	0.86	1.49	3.56
	d_{90} [min]	0.68	2.89	2.44	6.16
	$t_{T,median}$ [min]	2.75	7.41	6.48	**17.03**
	$t_{T,80}$ [min]	2.92	8.26	7.54	**19.76**
	$t_{T,90}$ [min]	3.43	10.29	8.49	**22.36**

Comparing the values of $t_{T,median}$, $t_{T,80}$, and $t_{T,90}$ (or other percentiles) with the mean travel time t_T, the reliability of the freeway route is clearly defined. The route under consideration has an average travel time of 18.40 min. The median of the travel time is 17.03 min. It is smaller than the average because of the positive skewness of the travel time distribution. The 90th percentile travel time is 23.36 min.

CONCLUSIONS

The travel time of a link or route can be considered as shifted Gamma distributed. Given the coefficient of variation of those distributions, the probability of trips on time and thus the reliability of travel time can be assessed. For estimating the coefficient of variation, the variance or standard deviation of travel time is required. For a whole route consisting of several individual links, the variance of the total route travel time can be calculated as a superposition of the variances of the individual links. Using the variance or standard deviation of travel time as an indicator of reliability, the reliability of a route or a network can be assembled from the reliability of the individual links. If the reliability (here represented by the variance or standard deviation of the travel time) for each type of road – empirically and theoretically – can be determined, the reliability of a route or a network can be easily estimated according to the proposed model. It was shown that the total standard deviation is not a linear but a concave (e.g. square) function of the total travel time. A model for estimating the standard deviation of travel time within a route and the parameters of the corresponding travel time distribution was presented. The application of the procedure for assessing travel time reliability of freeway networks was demonstrated for an example freeway route consisting of three links.

Statistical independence between the link travel times is assumed for simplification in the paper. This assumption is only required for links within a route and is not critical because a link normally has a significant length. The independence between sub-segments (which can be very short) within a link is not required and can be taken into account by parameters K, K_2 and K_3 of the proposed model.

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